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DEVELOPMENT AND APPLICATION
OF A SUBSONIC TRIANGULAR
VORTEX PANEL

THESIS

AFIT/GAE/AE/80J-1 John C. Sparks

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DEVELOPMENT AND APPLICATION
OF A SUBSONIC TRIANGULAR
VORTEX PANEL

THESIS

Presented to the Faculty of the School of Engineering

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by

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Preface

I would like to take this opportunity to express my thanks to my advisor, Major Stephen J. Koob of the Departmert of Aeronautics and Astronautics, Air Force Institute of Technology, for his guidance, support, and criticism during the preparation of this thesis. Thanks are also extended to James R. Snyder of ASD/XRHI for his valuable suggestions and to my wife, Carolyn, for putting up with me during the course of this study.

John C. Sparks

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List of Symbols

Symbol	
α	. . . angle of attack
$\vec{\omega}$	. . . vorticity vector
$\delta(x, y)$, $\gamma(x, y)$, 0	. . . components of $\vec{\omega}$
A, B, C	. . . coefficients of the assumed "bilinear" form for γ
D, E, F	. . . coefficients of the assumed "bilinear" form for δ
x_i, y_i	. . . panel corner points
δ_i, γ_i	. . . corner vorticity values
x, y, z	. . . chordwise, spanwise, and normal coordinates, respectively
Γ	. . . straight line segment
$\vec{s}$	. . . control point in the plane $z = 0$
u, v, w	. . . perturbation velocities in the respective x, y, and z directions
$\vec{i}$, $\vec{j}$, $\vec{k}$	. . . unit vectors in the respective x, y, and z directions
R	. . . region of integration
M	. . . number of chordwise panels or slope (Appendix A only)
N	. . . number of spanwise panels
Λ	. . . leading edge slope (Section III)
$[\theta_j]$	. . . column vector of corner vorticities
$[A_{ij}]$	. . . coefficient matrix premultiplying $[\theta_j]$
M_∞	. . . free stream Mach number
V_∞	. . . free stream velocity

List of Symbols

Symbol	
dc/dx	. . . camber slope
γ	. . . ratio of specific heats (eqs 3.30 and 3.31 only)
$f_i(x, y)$	. . . integrands obtained through application of the Biot-Savart Law
F_i	. . . improper integrals of the form

$$\lim_{L \to \infty} \int_{y_o}^{y_1} \int_{(y-b)/M}^{L} f_i(x, y)\,dx\,dy \quad i = 1, \text{—} 5$$

S_i	. . . strip function
I_i	. . . integral over a triangular region
C_P	. . . pressure coefficient
C_L	. . . local lift coefficient
C_M	. . . local pitching moment coefficient

Subscripts and Superscripts

L	. . . leading triangle
T	. . . trailing triangle

Abstract

Paneling methods are approximate techniques for solving flow problems over wings and bodies. Vortex panels are used to model flow over wings and other lifting surfaces. The author develops a triangular vortex panel having a vorticity distribution that can vary in magnitude and direction. This panel is used to predict the pressure distribution on a rectangular and a sweptback wing in subsonic flow. Lift distributions obtained compare favorably to Anderson's solution and wind tunnel results except near the wing tip. In this region, the distribution will spike before satisfying the Kutta condition imposed at the tip. Possible remedies for the tip problem are discussed.

DEVELOPMENT AND APPLICATION OF A SUBSONIC TRIANGULAR VORTEX PANEL

I. Introduction

Background

Paneling methods are approximate techniques for solving linearized subsonic and supersonic potential flow problems over wings and bodies. Panels in use today incorporate singularity distributions of the source, vortex, and doublet types. Source panels are often used to model bodies and other non-lifting surfaces and to model thickness of lifting surfaces. Vortex panels are used to model either lifting or non-lifting surfaces. In a typical problem, the airplane is represented by a finite number of panels. Each panel is a singularity distribution of unknown strength that models some part of the aerodynamic surface. These unknown strengths are determined by applying the flow tangency boundary conditions at or near the aerodynamic surfaces. Once the strengths are known, the perturbation velocities can be computed. These are substituted into the Bernoulli equation to obtain the pressure distribution and the corresponding aerodynamic forces and moments.

Problem Statement

In many current paneling methods, the orientation of the vorticity vector is fixed on the panel (Ref 6 and Ref 8). This leads to unacceptable errors in some cases. The purpose of this study is to derive a subsonic triangular panel having a vorticity distribution that can vary both in magnitude and direction. A panel system is assembled and used to predict the pressure distribution on a planform. Since the computations involved are too lengthy to be performed by hand, a computer code was developed to apply the panels to planar wings.

Approach

The subsonic triangular panel is derived in Section II. The Biot-Savart Law is used to compute the induced velocity at a point due to an assumed bilinear vorticity distribution on the panel. This expression formulates the induced velocity in terms of panel geometry and unknown corner point vorticity.

Section III presents methodology for panel system assembly. Numbering schemes are developed for panels and the unknown corner point vorticities. Planform boundary conditions are applied which reduce the number of unknowns and the remaining unknowns are solved for by formulating a linear system of equations. This system consists of control point equations (one per panel) and a number of edge continuity conditions. The linearized form of the flow tangency boundary condition is then used to effect the solution. Once the corner vorticities are known, the vorticity at any point on the planform can be obtained. Finally, induced velocities and corresponding pressures can be calculated from the known vorticity.

Section IV presents a computer code developed to apply the methodology to planar wings. Brief descriptions of each subroutine and a detailed input description are provided. Appendix B is a sample output and Appendix C is the program listing.

Section V presents program predictions for a rectangular wing which are compared against Anderson's solution (Ref 1:9-16). Predictions are also presented for a swept untapered wing which are compared against wind tunnel tests (Ref 4:92).

Section VI concludes the report and makes recommendations for the improvement of the aerodynamic model.

II. Panel Derivation

This section presents the development of the subsonic triangular panel. The goal is to derive an expression for the induced velocity at an arbitrarily chosen point in the xy plane due to an assumed bilinear vorticity distribution on the panel.

Geometry

The first step in panel development is the definition of panel geometry. Initially assume the panel is a trapezoid lying in the xy-plane and having two edges parallel to the x-axis. It is then subdivided into two triangles having a common side that joins the upper left hand corner to the lower right hand corner. The panel is oriented so that the root and tip chords lie parallel to the free stream flow direction at $\alpha = 0$. Figure 1 depicts the panel geometry, corner point numbering scheme, and coordinate system.

Singularity Strength Distribution

The general form of the singularity strength distribution on the panel will be

$$\vec{\omega}(x, y) = \delta(x, y)\vec{i} + \gamma(x, y)\vec{j} \qquad (2.1)$$

where δ and γ are continuous functions of x and y. For the purpose of this study, δ and γ are assumed to have the "bilinear" forms

$$\delta(x, y) = F + Dx + Ey \qquad (2.2)$$

$$\gamma(x, y) = A + Bx + Cy \qquad (2.3)$$

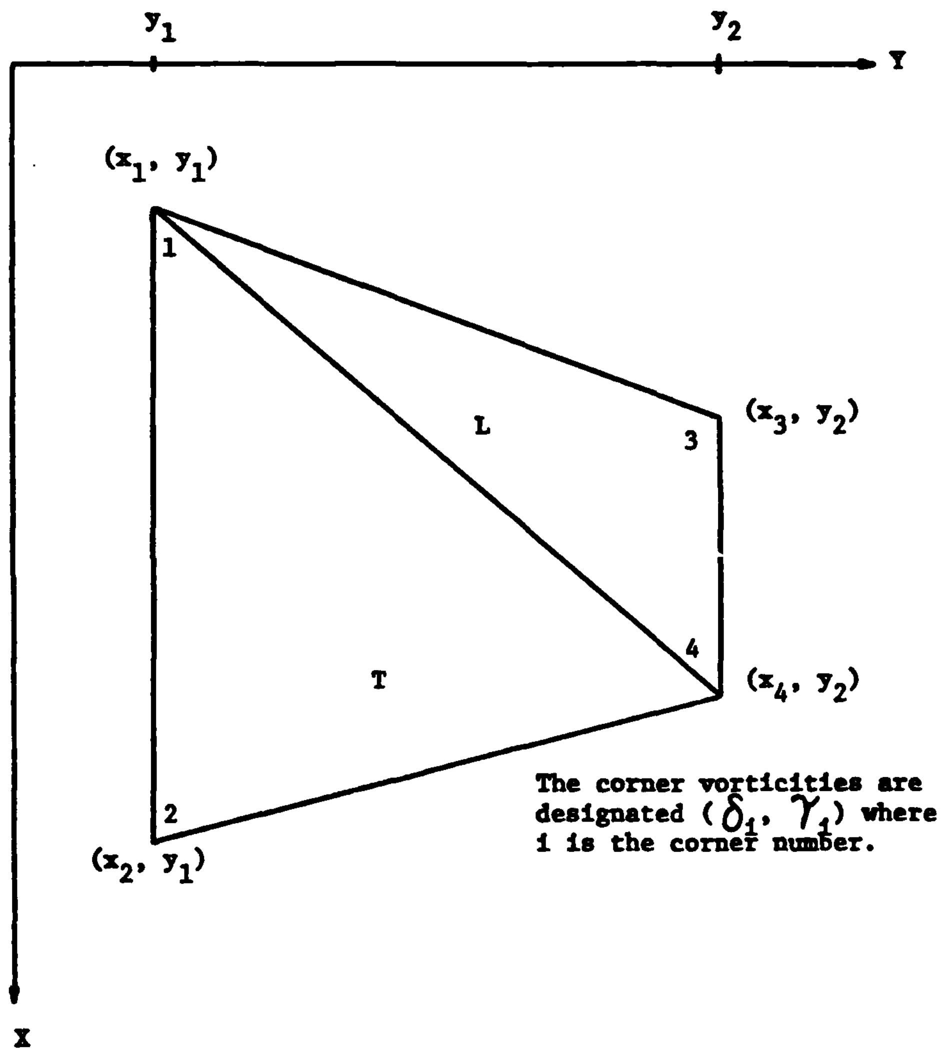

Figure 1. Panel Geometry and Corner Point Numbering Scheme

where the coefficients A, B, C, D, F, D and E are yet to be determined constants. These constants will be expressed as functions of the panel geometry and the unknown singularity strengths at the corner points.

Consequence of the Helmholtz Condition

The vorticity distribution $\vec{\omega}(x, y)$ is required to satisfy the Helmholtz condition that vorticity must be preserved in the fluid. Thus,

$$\nabla(\delta_j + \gamma_j) = 0 \tag{2.4}$$

on the panel. If δ and γ have the forms (2.2) and (2.3), respectively, then

$$\partial(F + Dx + Ey)/\partial x + \partial(A + Bx + Cy)/\partial y = 0 \tag{2.5}$$

which implies

$$D = -C \tag{2.6}$$

such that

$$\delta(x, y) = F - Cx + Ey \tag{2.7}$$

The formulation could be continued in terms of the bilinear coefficients. However, it is conventional to express them in terms of panel corner point vorticities.

Bilinear Coefficients in Terms of Corner Vorticity

In the ensuing discussion, the subscript L refers to the leading triangle and the subscript T refers to the trailing triangle.

The γ component of vorticity (eq (2.3)) is assigned the unknown values γ_1, γ_3, γ_4 at the corners (x_1, y_1), (x_3, y_2), and (x_4, y_2) of the leading triangle. This leads to the following system of three equations in A_L, B_L and C_L:

$$\gamma_1 = A_L + B_L x_1 + C_L y_1 \tag{2.8}$$

$$\gamma_3 = A_L + B_L x_3 + C_L y_2 \tag{2.9}$$

$$\gamma_4 = A_L + B_L x_4 + C_L y_2 \tag{2.10}$$

The system has the solution:

$$\begin{aligned} A_L = y_2\ \gamma_1/(y_2 - y_1) &+ [(y_1 x_4 - x_1 y_2)/(y_2 - y_1)] \\ &\gamma_3/(x_3 - x_4) + [(x_1 y_2 - y_1 x_3)/(y_2 - y_1)] \\ &\gamma_4/(x_3 - x_4) \end{aligned} \tag{2.11}$$

$$B_L = (\gamma_3 - \gamma_4)/(x_3 - x_4) \tag{2.12}$$

$$\begin{aligned} C_L = -\ \gamma_1/(y_2 - y_1) &+ [(x_1 - x_4)/(y_2 - y_1)]\ \gamma_3/ \\ &(x_3 - x_4) + [(x_3 - x_1)/(y_2 - y_1)]\ \gamma_4/ \\ &(x_3 - x_4) \end{aligned} \tag{2.13}$$

Since $D_L = - C_L$, only two corner conditions may be used to solve for F_L and E_L in the δ component eq (2.7). Assigning the values δ_1 and δ_3 at the corners (x_1, y_1) and (x_3, y_2) leads to

$$\delta_1 + C_L x_1 = E_L y_1 + F_L \tag{2.14}$$

$$\delta_3 + C_L x_3 = E_L y_2 + F_L \tag{2.15}$$

and

$$E_L = (\delta_1 - \delta_3)/(y_1 - y_2) + [(x_1 - x_3)/(y_1 - y_2)]C_L \tag{2.16}$$

$$F_L = (y_1 \delta_3 - y_2 \delta_1)/(y_1 - y_2) + [(y_1 x_3 - x_1 y_2)/(y_1 - y_2)]C_L \tag{2.17}$$

In a similar way, the trailing triangle coefficient equations are obtained as follows:

$$A_T = y_1 \gamma_4/(y_1 - y_2) + [(y_2 x_1 - x_4 y_1)/(y_1 - y_2)] \gamma_2/(x_2 - x_1) + [(x_4 y_1 - y_2 x_2)] \gamma_1/(x_2 - x_1) \tag{2.18}$$

$$B_T = (\gamma_2 - \gamma_1)/(x_2 - x_1) \tag{2.19}$$

$$C_T = -\gamma_4/(y_1 - y_2) + [(x_4 - x_1)/(y_1 - y_2)] \gamma_2/(x_2 - x_1) + [(x_2 - x_4)/(y_1 - y_2)] \gamma_1/(x_2 - x_1) \tag{2.20}$$

$$E_T = (\delta_2 - \delta_4)/(y_1 - y_2) + [(x_2 - x_4)/(y_1 - y_2)]C_T \tag{2.21}$$

$$F_T = (y_1 \delta_4 - y_2 \delta_2)/(y_1 - y_2) + [(y_1 x_4 - x_2 y_2)/(y_1 - y_2)]C_T \quad (2.22)$$

where E_T and F_T have been expressed in terms of (as functions of) $\vec{\omega}$ components at corners 2 and 4.

Mathematical Continuity

The bilinear vorticity distribution is continuous on each triangular region. In addition, the γ component has been made to be continuous throughout the planform by the representation in terms of corner values. This can be demonstrated as follows. Let Γ be the boundary shared by any two adjacent triangles. Then Γ is a straight line segment and can be described by a linear expression (i.e., y in terms of x or x in terms of y). The γ distribution on each of the adjacent triangles will degenerate to a linear function of a single variable upon substitution of this expression. Both functions assume the same γ values at the endpoints of Γ. Since only two points are needed to determine a straight line or a linear form, we have γ matching identically on Γ.

The δ component has breaks in continuity throughout the planform. This is a consequence of applying the Helmholtz condition (eq (2.4)) which eliminated the constant D and expressing the remaining two unknowns in terms of two δ corner values, out of a possible three. The δ component is continuous on panel leading and trailing edges since it is on these edges that common δ values are assumed at the endpoints. The δ component is discontinuous on panel diagonals.

Application of the Biot-Savart Law

Let $\vec{\omega}(\vec{x})$ be a vorticity distribution defined on a finite region R in the xy plane. Let $\vec{s}$ be a fixed point (control point) in the plane. The velocity induced at $\vec{s}$ due to the distribution on R is given by the Biot-Savart Law (Ref 5:526-528):

$$4\pi\vec{V}(\vec{s}) = \iint_R [\,\vec{\omega}(\vec{x}) \times (\vec{s} - \vec{x})]/|\vec{s} - \vec{x}|^3 dR \qquad (2.23)$$

Suppose the control point $\vec{s}$ is located at the origin of the coordinate system. Note that this can be done by performing a simple translation of the plane. Then,

$$\vec{s} = (0,\ 0,\ 0) \qquad (2.24)$$

$$\vec{s} - \vec{x} = (0,\ 0,\ 0) - (x,\ y,\ 0) = (-x,\ -y,\ 0) \qquad (2.25)$$

Assuming the distribution has the form (2.1),

$$\vec{\omega} \times (\vec{s} - \vec{x}) = [x\gamma - y\delta\,]\vec{k} \qquad (2.26)$$

where $\vec{k}$ is the unit vector normal to the xy-plane. Substituting the expressions (2.25) and 2.26) into eq (2.23) yields:

$$4\pi w\ (0,\ 0,\ 0) = \iint_R (x\gamma - y\,\delta\,)/(x^2 + y^2)^{3/2} dR \qquad (2.27)$$

where w is the normal velocity component induced at the origin. Substituting the expressions for γ (2.3) and δ (2.7) into eq (2.27) yields:

$$4\pi w = \iint_R (Ax + Bx^2 + 2Cxy - Ey^2 - Fy)/$$

$$(x^2 + y^2)^{3/2} dR \qquad (2.28)$$

where R is taken as the region defined by a trapezoidal panel.

Let R_L and R_T be the subregions of R which correspond to the leading and trailing triangles. The coefficients A, B, C, F, and E remain constant on each subregion and eq (2.28) is rewritten as:

$$4\pi w = A_L I_1^L + A_T I_1^T + B_L I_2^L + B_T I_2^T + 2C_L I_3^L +$$

$$2C_T I_3^T - F_L I_4^L - F_T I_4^T - E_L I_5^L - E_T I_5^T \qquad (2.29)$$

where

$$I_3^{L,\ T} = \iint_{R_{L,\ T}} (x/(x^2 + y^2)^{3/2}) dR \qquad (2.30)$$

$$I_2^{L,\ T} = \iint_{R_{L,\ T}} (x^2/(x^2 + y^2)^{3/2}) dR \qquad (2.31)$$

$$I_3^{L,\ T} = \iint_{R_{L,\ T}} (xy/(x^2 + y^2)^{3/2}) dR \qquad (2.32)$$

$$I_4^{L,\ T} = \iint_{R_{L,\ T}} (y/(x^2 + y^2)^{3/2}) dR \qquad (2.33)$$

$$I_5^{L,\ T} = \iint_{R_{L,\ T}} (y^2/(x^2 + y^2)^{3/2}) dR \qquad (2.36)$$

Evaluation of these integrals may be found in Appendix A. Substitution of the expressions for A_L through E_T and collecting coefficients of the unknowns (δ_1, δ_2, δ_3, δ_4, γ_1, γ_2, γ_3, and γ_4) yields, after considerable algebraic manipulation,

$$
\begin{aligned}
4\pi w = {} & [(y_2 I_4^L - I_5^L)/(y_1 - y_2)]\ \delta_1 + \\
& [(y_2 I_4^T - I_5^T)/(y_1 - y_2)]\ \delta_2 + \\
& [(I_5^L - y_1 I_4^L)/(y_1 - y_2)]\ \delta_3 + \\
& [(I_5^T - y_1 I_4^T)/(y_1 - y_2)]\ \delta_4 + \\
& [((x_4 y_1 - y_2 x_2) I_1^T - (x_2 - x_1) y_2 I_1^L - \\
& (y_1 - y_2) I_2^T + (x_2 - x_1) K_L + \\
& (x_2 - x_4) K_T)/((x_2 - x_1)(y_1 - y_2))]\ \gamma_1 + \\
& [((y_2 x_1 - x_4 y_1) I_1^T + (y_1 - y_2) I_2^T + \\
& (x_4 - x_1) K_T)/((y_1 - y_2)(x_2 - x_1))]\ \gamma_2 + \\
& [((y_1 x_4 - x_1 y_2) I_1^L + (y_2 - y_1) I_2^L + \\
& (x_1 - x_4) K_L)/((y_2 - y_1)(x_3 - x_4))]\ \gamma_3 + \\
& [((x_1 y_2 - y_1 x_3) I_1^L - (x_3 - x_4) y_1 I_1^T - \\
& (y_2 - y_1) I_2^L + (x_3 - x_1) K_L + (x_3 - x_4) K_T)/
\end{aligned}
$$

$$((y_2 - y_1)(x_3 - x_4))] \gamma_4 \tag{2.35}$$

where

$$K_L = 2I_3^L - [(x_1 - x_3)/(y_1 - y_2)]I_5^L -$$

$$[(y_1x_3 - x_1y_2)/(y_1 - y_2)]I_4^L \tag{2.36}$$

$$K_T = 2I_3^T - [(x_2 - x_4)/(y_1 - y_2)]I_5^T -$$

$$[(y_1x_4 - x_2y_2)/(y_1 - y_2)]I_4^T \tag{2.37}$$

<u>Summary</u>

Expression (2.35) is the normal velocity induced at the origin of the xy plane by a trapezoidal vorticity panel. This velocity is due to a bilinear vorticity distribution which satisfies the Helmholtz condition eq (2.4).

III. Panel Assembly

This section presents the panel assembly procedures needed to model a wing. The goal is to develop the methodology required to predict the pressure distribution and associated forces and moments on a wing.

Panel Numbering

Figure 2 illustrates a paneling arrangement and associated numbering scheme for a 16 panel wing. The panels are numbered consecutively in the chordwise direction starting with the inboard leading edge panels and terminating with the outboard trailing edge panels.

Number of Unknowns, Boundary Conditions and Numbering

Let M be the number of chordwise panels and N be the number of spanwise panels. These are defined using M + 1 chordwise cuts and N + 1 spanwise cuts. Each intersection determines a panel corner point. Since there are two unknown components at each corner point, the total number of unknowns is given by:

$$2(M + 1)(N + 1) \tag{3.1}$$

Boundary Conditions

Two boundary conditions are imposed on the wing panel system. These reduce the number of unknowns and improve the physical modeling of the flow field.

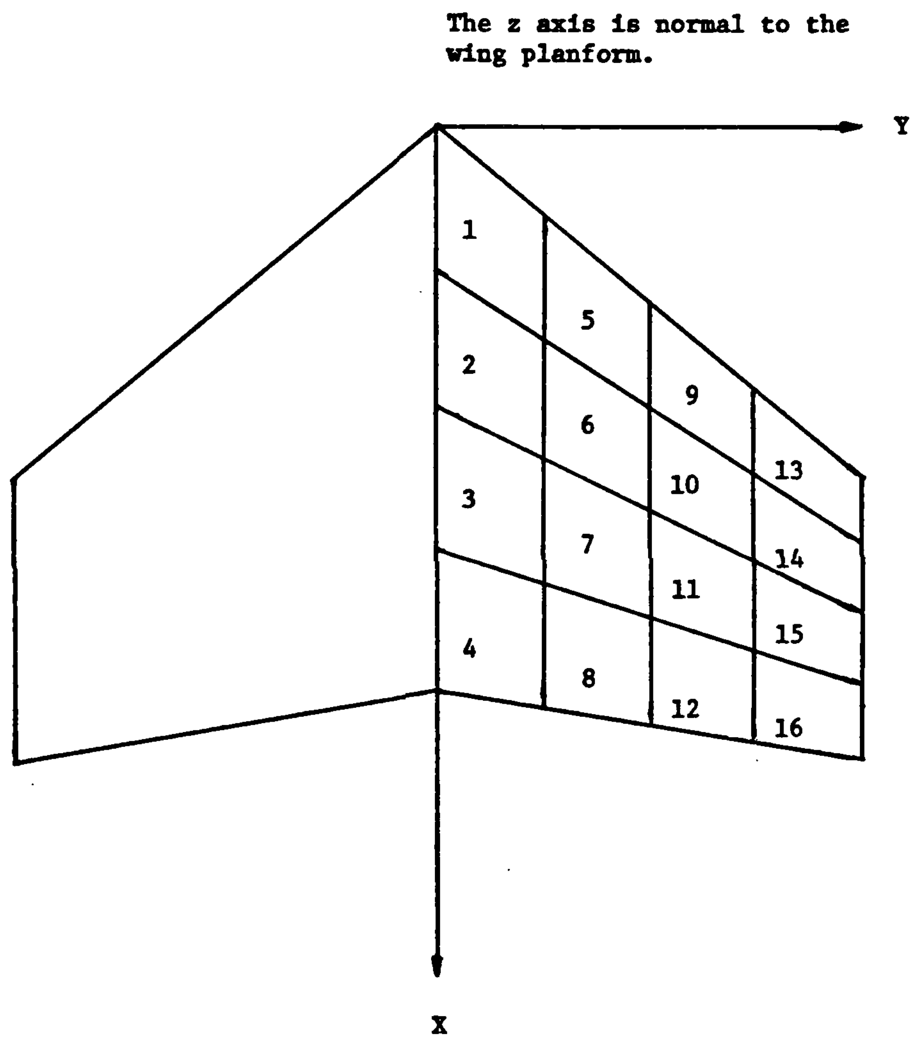

Figure 2. A Paneling Arrangement and Associated Numbering Scheme for a 16 Panel Wing

The Kutta condition (Ref 5:390-399)

$$\gamma(x, y) = 0 \tag{3.2}$$

is imposed at all corner points on the wing trailing and tip edges. Since the γ component of vorticity eq (2.2) is both linear and continuous on these edges, the Kutta condition is satisfied identically. The Kutta condition reduces the total number of unknowns by:

$$M + N + 1 \tag{3.3}$$

(NOTE: The point defining the intersection of the trailing and tip edges is common to both.)

A historically acceptable boundary condition (an outgrowth of Prandtl's *lifting-line theory* (Ref 5:535-567)) is for the vorticity vector to lie tangent to the wing leading edge. This boundary condition initially orients the vorticity vector so that a positive circulation is produced. The boundary condition is imposed at all leading edge corner points and can be written:

$$\gamma/\delta = \Lambda \tag{3.4}$$

or

$$\delta = \gamma/\Lambda \tag{3.5}$$

where Λ is the leading edge slope at the corner point. It reduces the total number of unknowns by:

$$N + 1 \tag{3.6}$$

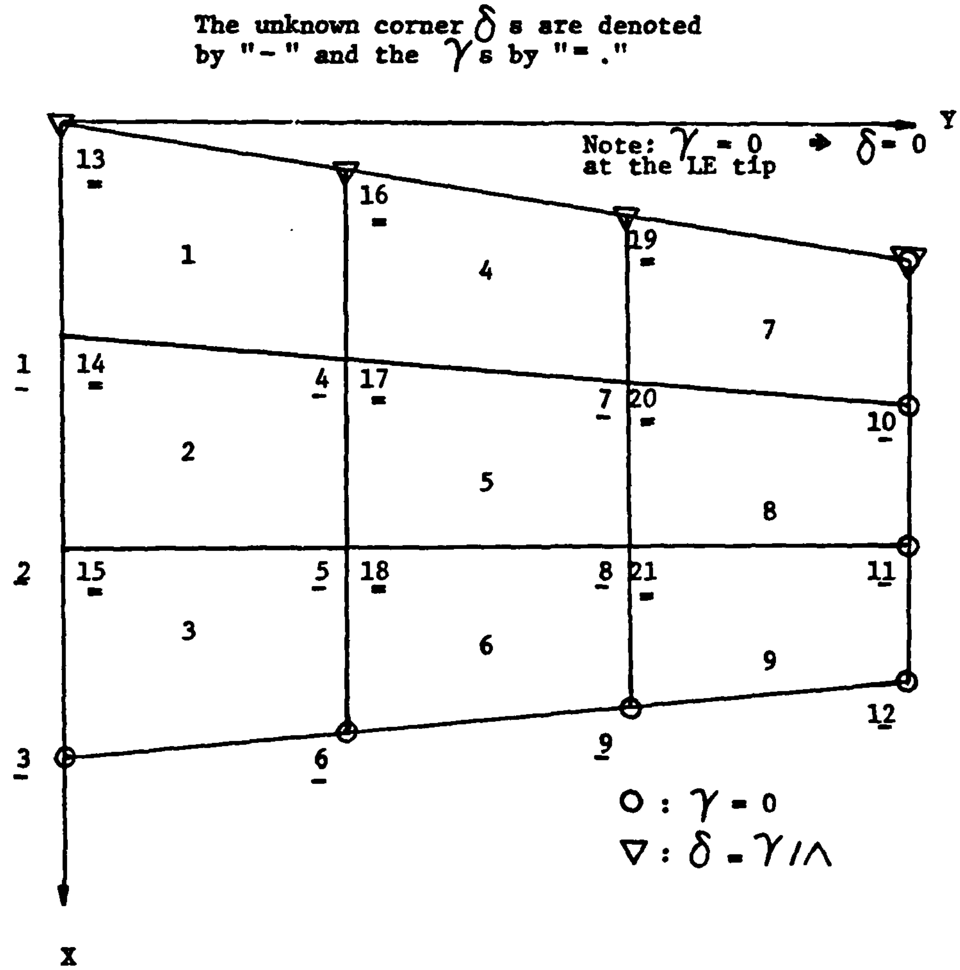

Figure 3. Unknown Numbering Scheme for a 9 Panel Wing

and the total number of unknowns becomes:

$$2MN + M \tag{3.7}$$

after imposing the two boundary condition equations (3.2) and (3.5).

Since the wing is symmetric about the x-axis, it may seem logical to impose a boundary condition at the wing root chord. However, setting

$$\delta = 0 \tag{3.8}$$

at the centerline is redundant for rectangular wings and leads to an ill-conditioned system once planform symmetry is considered.

Unknown Numbering Scheme

Figure 3 illustrates the unknown numbering scheme for a 9 panel wing with applied boundary conditions. The paneling arrangement of Figure 3 is chosen because it represents the smallest number of panels needed to illustrate interior panels and panels having boundary conditions. Let i be the panel number of an interior panel. Then the following numbers (in terms of i, M, and N) are assigned to the eight unknown corner vorticity components:

$$(\delta_1) \quad i - 1 \tag{3.9}$$

$$(\delta_2) \quad i \tag{3.10}$$

$$(\delta_3) \quad i + M - 1 \tag{3.11}$$

$$(\delta_4) \quad i + M \tag{3.12}$$

$$(\gamma_1) \quad MN + M + i \tag{3.13}$$

$$(\gamma_2) \quad MN + M + i + 1 \tag{3.14}$$

$$(\gamma_3) \quad MN + 2M + i \tag{3.15}$$

$$(\gamma_4) \quad MN + 2M + i + 1 \tag{3.16}$$

Solving for the Corner Vorticities

The total number of unknowns (after the boundary conditions are applied) is given by eq (3.7) which also specifies the number of conditions needed to solve for the corner vorticities. Two types of conditions will be used; control point conditions and edge or point continuity conditions.

Control Point Equations

Control point equations are obtained using eq (2.35). The velocity component is computed for one point (control point) on each panel comprising the wing.

Let (x_i, y_i) be the control point on panel i. To obtain the contribution to w_i due to panel j, express the coordinates of panel j in a coordinate system with (x_i, y_i) at the origin. This is done by performing a translation in the z = o plane:

$$\begin{aligned} x' &= x - x_i \\ y' &= y - y_i \end{aligned} \tag{3.17}$$

Equation (2.35) is then applied with appropriate boundary conditions.

Planform symmetry is included by reflecting either the panel or control point about the x-axis and applying eq (2.35). Reflecting the control point is less complicated from a programming viewpoint.

The above process is repeated for each panel on the wing. After all the contributions to w_i have been calculated, it can be written as:

$$w_i = \sum_{j=1}^{2MN+M} A_{ij}\theta_j \tag{3.18}$$

where θ_j is the column vector of unknown corner vorticities and the coefficients A_{ij} are functions of panel geometry. The θ_j are numbered using the system given by eqs (3.9) through (3.16). One control point equation (3.18) is obtained for each panel on the wing and together they comprise MN conditions.

Edge Continuity Conditions

The δ component of vorticity eq (2.7) is discontinuous across the panel diagonal (See discussion in Section II.). This can be partially remedied by specifying a point continuity condition at the panel lower right hand corner. This condition is:

$$\delta_L(x_4, y_2) = \delta_4 \tag{3.19}$$

which becomes

$$\begin{aligned}&\delta_4 - \delta_3 + [(x_3 - x_4)/(y_2 - y_1)]\,\gamma_1 - \\ &[(x_1 - x_4)/(y_2 - y_1)]\,\gamma_3 - [(x_3 - x_1)/ \\ &(y_2 - y_1)]\,\gamma_4 = 0\end{aligned} \tag{3.20}$$

after substituting of (x_4, y_2) and the expressions for A_L (2.11), B_L (2.12), and C_L (2.13) into eq (2.7). One condition eq (3.20) is formulated for each panel on the wing for a total of MN conditions. Note that δ is still not continuous across the panel diagonal due to the remaining discontinuity at the upper left hand corner. Also, δ is not continuous across panel side edges.

The edge continuity conditions and control point equations total 2MN conditions. M additional conditions can be obtained by specifying an edge continuity condition for δ at the upper left hand corner of each panel along the centerline. This condition is:

$$\delta_T(x_1, y_1) = \delta_1 \tag{3.21}$$

which becomes

$$\begin{aligned} &\delta_2 - \delta_1 - [(x_2 - x_1)/(y_1 - y_2)]\,\gamma_4 + \\ &[(x_4 - x_1)/(y_1 - y_2)]\,\gamma_2 + [(x_2 - x_4)/ \\ &(y_1 - y_2)\,\gamma_1 = 0 \end{aligned} \tag{3.22}$$

after substitution of (x_1, y_1) and the expressions for A_T (2.18), B_T (2.19), and C_T (2.20) into eq (2.7). This choice is based on trial and error, the additional δ continuity on the centerline having the effect of minimizing vorticity oscillations.

Compressibility

Compressibility effects are accounted for by using the Prandtl-Glauert transformation (Ref 2:124-127):

$$\bar{x} = x/\sqrt{1 - M^2} \tag{3.23}$$

The transformation is applied to all x coordinates which are used in either the control point equations or edge continuity conditions.

Matrix Formulation

The control point equations and edge continuity conditions are 2MN + M equations in the unknowns, θ_j. This system has the matrix formulation:

$$\underset{(2MN + M)x(2MN + M)}{\begin{bmatrix} A_{ij} \end{bmatrix}} \cdot \begin{bmatrix} \theta_1 \\ \vdots \\ \theta_{2MN + M} \end{bmatrix} = \begin{Bmatrix} w_1 \\ \vdots \\ w_{MN} \\ O \end{Bmatrix} \tag{3.24}$$

The first MN rows of A_{ij} are the coefficients for the control point equations and are all nonzero. The last MN + M rows of A_{ij} correspond to the homogeneous edge continuity conditions and have no more than five nonzero entries per row.

Solution

The linearized form of the flow tangency boundary condition is (Ref 5:495):

$$w/V_\infty = dc/dx - \alpha \tag{3.25}$$

where α is the wing angle of attack, dc/dx is the local camber slope and V_∞ is the free stream velocity. This expression is substituted for each w_i in eq 3.24 where dc_i/dx is the panel slope at control point i. In matrix notation,

$$\left[A_{ij}\right] \cdot \left[\theta_j\right] = \left\{ \begin{matrix} V_\infty (dc_i/dx - \alpha) \\ 0 \end{matrix} \right\} \tag{3.26}$$

or

$$\theta_j / V_\infty = \left[A_{ij}\right]^{-1} \cdot \left\{ \begin{matrix} dc_i/dx - \alpha \\ 0 \end{matrix} \right\} \tag{3.27}$$

Forces and Moments

Once the θ_j / V_∞ are obtained for a given α and camber slope distribution, eqs (2.2) and (2.7) can be used to calculate the vorticity strength at any point on the planform. The surface perturbation velocities in terms of local vorticity are (Ref 5:508):

$$u/V_\infty = \pm \gamma/2 \tag{3.28}$$

$$v/V_\infty = \mp \delta/2 \tag{3.29}$$

The upper sign corresponds to the upper wing surface and visa-versa.

Pressure coefficients are obtained from the perturbation velocities by either using the exact isentropic expression (Ref 3:167):

$$C_p = 2[(1 + (\gamma - 1)M_\infty^2/2(1 - ((V_\infty + u)^2 + v^2 + w^2)/V_\infty^2)^{\gamma/(\gamma - 1)} - 1]/(\gamma M_\infty^2) \tag{3.30}$$

or the second order approximation (Ref 3:167):

$$C_p = -[2u/V_\infty + (1 - M_\infty^2)u^2/V_\infty^2 + (v^2 + w^2)/V_\infty^2] \tag{3.31}$$

which is adequate for two-dimensional and planar flows. These coefficients are integrated along chord lines to obtain local lift and moment coefficients. The appropriate expressions are

$$C_L = (1/c) \int_{X_{LE}}^{X_{TE}} (C_{p_l} - C_{p_u}) dx \tag{3.32}$$

$$C_M = (1/c^2) \int_{X_{LE}}^{X_{TE}} (C_{p_l} - C_{p_u}) x \, dx \tag{3.33}$$

where the subscripts l and u refer to the lower and upper wing surfaces.

This concludes the theoretical section of this report. The next step is the implementation of this panel to predict airloads on wings. This is done by the use of a computer code "WING" which is presented in Section IV.

IV. Computer Code

General Description

A FORTRAN code "WING" has been developed to analyze planform flow using the methodology discussed in Sections II and III. WING is a pilot code and should not be treated as a fully checked-out production program until the problems in the wing tip region are resolved (Section V). Many of the programming techniques used in WING have been previously developed by the author and can be found in Reference 8. Care has been taken to insure correspondence between FORTRAN variable names and the symbol usage in Sections II and III. The listing (Appendix C) contains comment cards that describe program function and logic in detail. Below is a brief description of each of the subroutines in WING.

WING (Main)

WING is the executive control routine. All geometric data is read by WING. WING initializes panel parameters and calls subroutines MESH, AERO, INVRT, and LOADS in that order.

MESH

MESH generates the x and y coordinates for the panel corner points and control points. The mesh is generated from the information given on the first four data cards. MESH is a FORTRAN version of the mesh generator discussed in Reference 8.

AERO

AERO formulates the control point equations and edge continuity conditions. Subroutine INT is called by AERO.

INT

INT evaluates the panel integrals using equations (A.31) and (A.32). Subroutine STRIP is called by INT.

STRIP

STRIP evaluates the five STRIP function equations (A.24) through (A.28) given two points in the plane.

INVRT

INVRT inverts the coefficient array for the system of equations formulated by AERO. The inversion is performed using Gaussian elimination. INVRT is essentially the same inversion subroutine found in the FASTLODS lifting surface program (Ref 6:73-133).

LOADS

LOADS calculates the planform pressure distributions and aerodynamic coefficients for the loading cases specified by cards 6 and 7. After the last loading case is examined, LOADS will terminate program execution.

Input Description

This section provides a card by card description of input data along with some helpful "dos and don'ts" of program operation. All input is unformatted and should be separated by commas. Integer data cannot have a decimal point. WING is a nondimensional code which will accept data in any consistent system of units. Presently, WING can analyze planforms having sixty panels or less. This can be increased by changing the dimensions of arrays X, Y, XC, YC, E, A, SG, CBR, and SUM in common blocks A and C. The reader is referred to Appendix B which contains a sample problem.

<u>Card 1 (Span Data)</u>

Variables (In Order)	Description
SSPN	The length of the wing semi-span.
NS	The number of stations needed to define the spanwise panel boundaries. The wing root chord is station 1 and the last station is the wing tip. NS is an integer and must be less than or equal to 12.
S(1), ..., S(NS)	Span stations as a fraction of the semi-span. 0. and 1. will always be the first and last entries. Entries must be in ascending order left to right.

Card 2 (Chord Data)

NC	The number of stations needed to define the chordwise panel boundaries The leading edge is station 1 and the last station is the trailing edge. NC is an integer and must be less than or equal to 10.
C(1), ..., C(NC)	Chord stations as a fraction of the wing local chord. 0. and 1. will always be the first and last entries. Entries must be in ascending order left to right.

Card 3 (Break Point Data)

Break points are the x coordinates of leading and trailing edge for those chords that define a sweep change. The wing centerline defines the positive x-axis with origin at the leading edge.

NB	The number of break point sets needed to outline the planform geometry. Two sets will be needed to define a four-point wing. NB is an integer and must be less than or equal to 10.

NL(1), B(1, 1), B(1, 2), ..., NL(NB), B(NB, 1), B(NB, 2)	NL is the number (Not the value!) of the span station where the sweep change occurs. NL is an integer. B(I, 1) and B(I, 2) are the leading and trailing edge x coordinates of the chord line at span station NL(I). Break point sets are entered from inboard to outboard. The first set (NL = 1) will always be the x coordinates of the root chord leading and trailing edge. The last set (NL = NS) will always be the x coordinates of the tip chord leading and trailing edge.

Card 4 (Mach and Control Point Data)

CY, CX	Control point location in terms of local panel span and chord. CY is the fraction of the panel span and CX is the fraction of the panel chord. A recommended control point choice is CY = .15 and CX = .75 which is based on extensive program testing.
MACH	Mach number (not an integer). WING accepts subsonic Mach numbers only.

Card 5 (Number of Loading Cases)

NA	The number of loading cases to be examined - i.e., changes in angle of attack or camber slope distribution. NA is an integer that has no upper bound.

Card 6 (Load Cases)

NA cards are required

ALPHA	The angle of attack in degrees. Plus is nose up.
NCCG	The camber change parameter (integer). Enter 0 to read a new camber slope distribution or 1 to retain the previous distribution. WING initially sets the camber slope array equal to 0.
NPF	The pressure option parameter (integer). Enter 0 to use the exact isentropic expression eq (3.30) or enter 1 to use the 2nd order approximation eq (3.31).

CY, CX	Panel location (fraction of panel span, fraction of panel chord) where the pressures are to be calculated. Pressures may be computed at points other than the control points.

Card 7 (Camber Slope Distribution)

This card is used only for a NCCG value of 0.

CBR(1) ..., CBR(NP)	The value of the local panel slope in degrees for each panel on the planform. NP is the total number of panels. Entries must be made in the order corresponding to the panel numbering scheme (Figure 2). The sign rule for camber follows the standard convention.

V. Results

Program WING was exercised for a variety of four point wings having various aspect ratios, taper ratios, and sweep angles. This section presents results for two of these wings.

Two general observations are made first. One, control point location is the major factor controling bounded numerical oscillations of the vorticity vector as it is in many current paneling routines (ex. Refs 6 and 8). Oscillations are very common if the control point is located anywhere on the leading triangle. Fewer oscillations occur if the control point is located on the trailing triangle with $.1 \le CY \le .5$ and $.4 \le CX \le .9$. Secondly, the program shows the best results when uniform spanwise paneling is used. Non-uniform spanwise paneling tends to cause oscillations in the vorticity vector. However, non-uniform chordwise paneling seems to have little effect on solution stability. The best total C_L match (with other known solutions) occurs at approximately $CY = .15$ and $CX = .75$.

Rectangular Wing

The first case examined is a rectangular wing; $AR = 8$, $\alpha = 5^\circ$ and $M_\infty = .1$. The wing is modeled using 12 uniformly spaced span stations and 6 non-uniformly spaced chord stations (0., .1, .3, .6, .8, and 1.) which define 55 panels. Figure 4 shows the C_L distribution predicted by WING and Anderson (Ref 1:9-16). The lift coefficient experiences a spiking phenomena near the wing tip. This phenomena always happens in

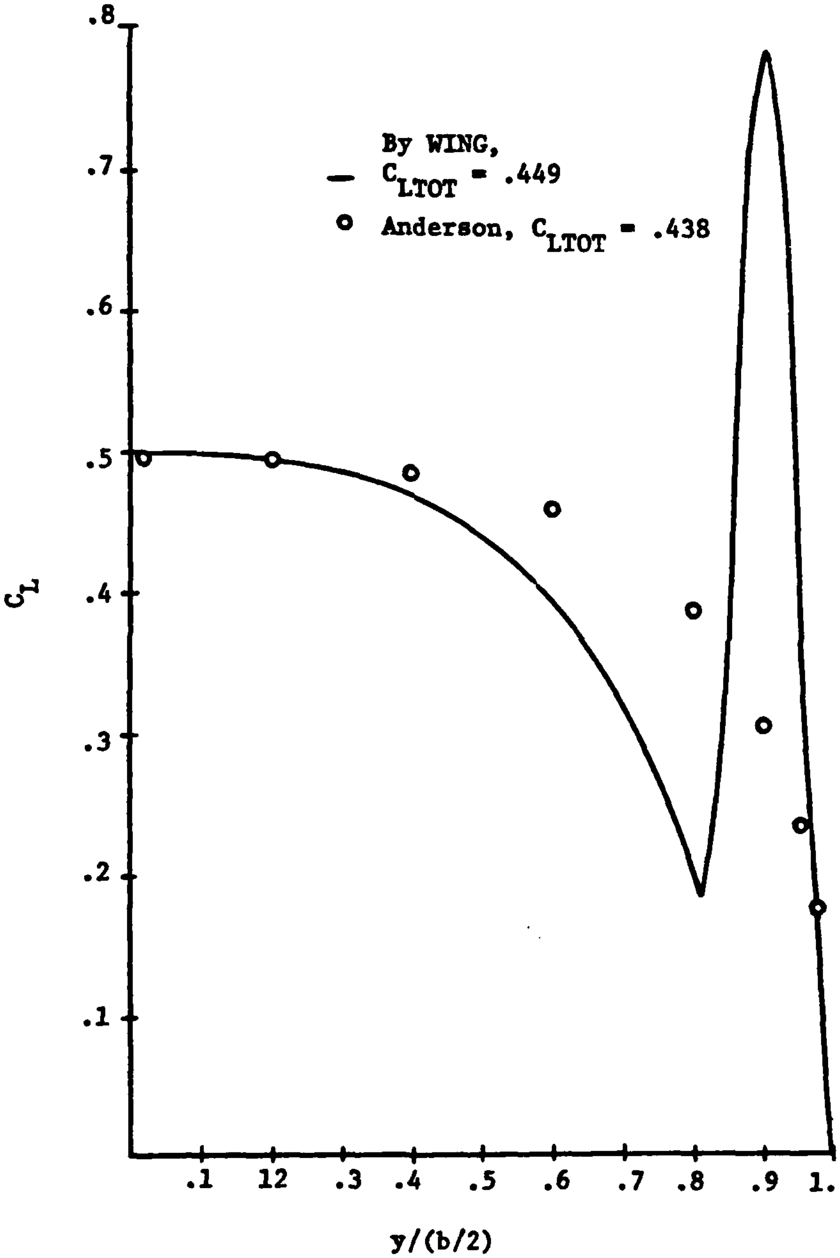

Figure 4. C_L Versus Span Station for a Rectangular Wing; AR = 8, $\alpha = 5^{\circ}$

the region defined by the last three outboard span stations. Also, the C_L distribution falls off faster than it should before reaching the region where spiking occurs. There is very little difference between total lift coefficients indicating the areas under the curves are approximately equal. This shows the solution is possibly trying to compensate for the spike by underpredicting lift in the inboard region.

Figure 5 shows the spanwise distribution of center of pressure. The X_{cp} shifts are aft in the region of spiking before traveling forward. The ΔCp versus chord station curves of Figure 6 exhibit expected behavior for the inboard stations ($0 = Y/S = .585$). At $Y/S = .9$, the curve has "fattened up" considerably which drives the X_{cp} backwards and creates the C_L spike. The curve is subsiding again at $Y/S = .95$ since the Kutta (no net load) boundary condition is imposed at the tip.

Figure 7 is the γ strength at the root chord. The calculated solution compares favorably with the exact 2-D flat plate solution (Ref 5:515):

$$\gamma(x) = 2\alpha\,[(c - x)/(cx - x^2)] \tag{5.1}$$

where c is the chord length and α is measured in radius.

Figures 8 and 9 show γ and δ strength distributions at selected span stations. The δ distribution grows in magnitude relative to γ as we approach either the tip or the trailing edge. This allows the vorticity vector to turn and satisfy the Kutta condition as shown in Figure 10. The δ component is 0 at the leading edge which

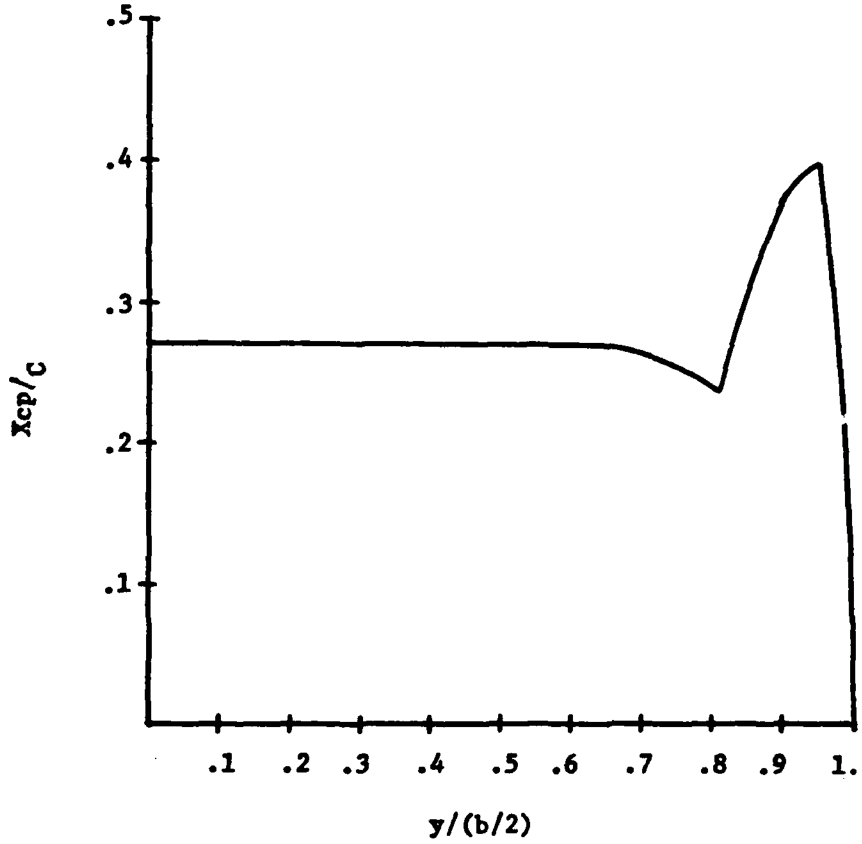

Figure 5. $Xcp/_C$ Versus Span Station for a Rectangular Wing; AR = 8, α = 5°

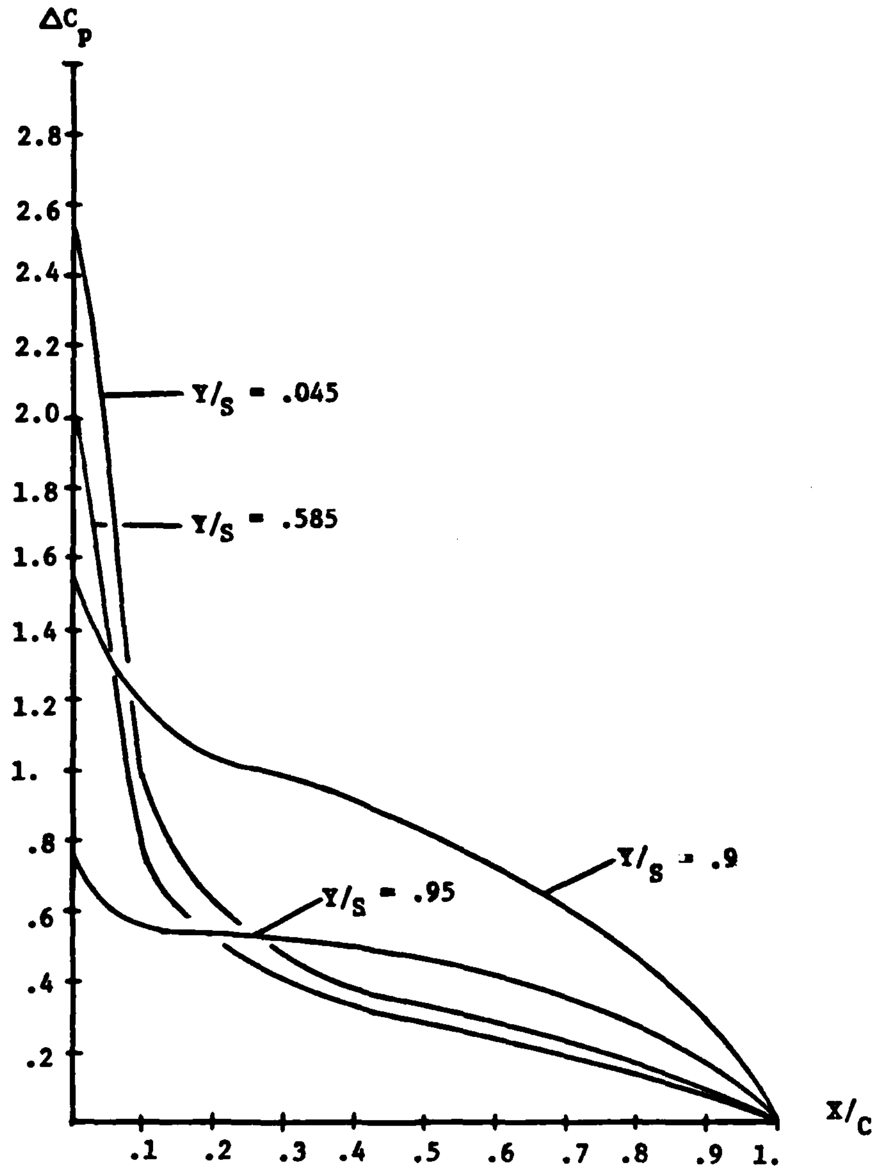

Figure 6. ΔC_p Versus $X/_C$ at Selected Span Stations - Rectangular Wing, AR = 8, α = 5°

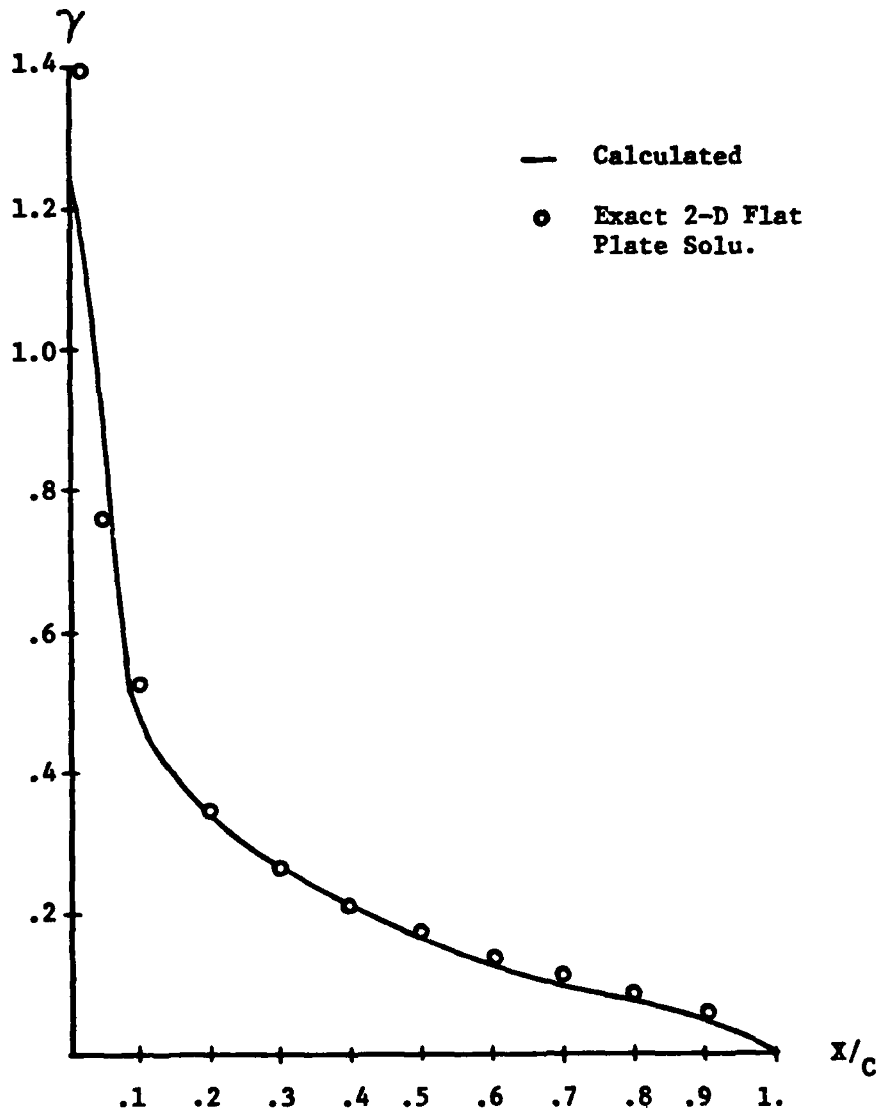

Figure 7. γ Strength Distribution at the Root Chord of a Rectangular Wing; AR = 8, $\alpha = 5^{\circ}$

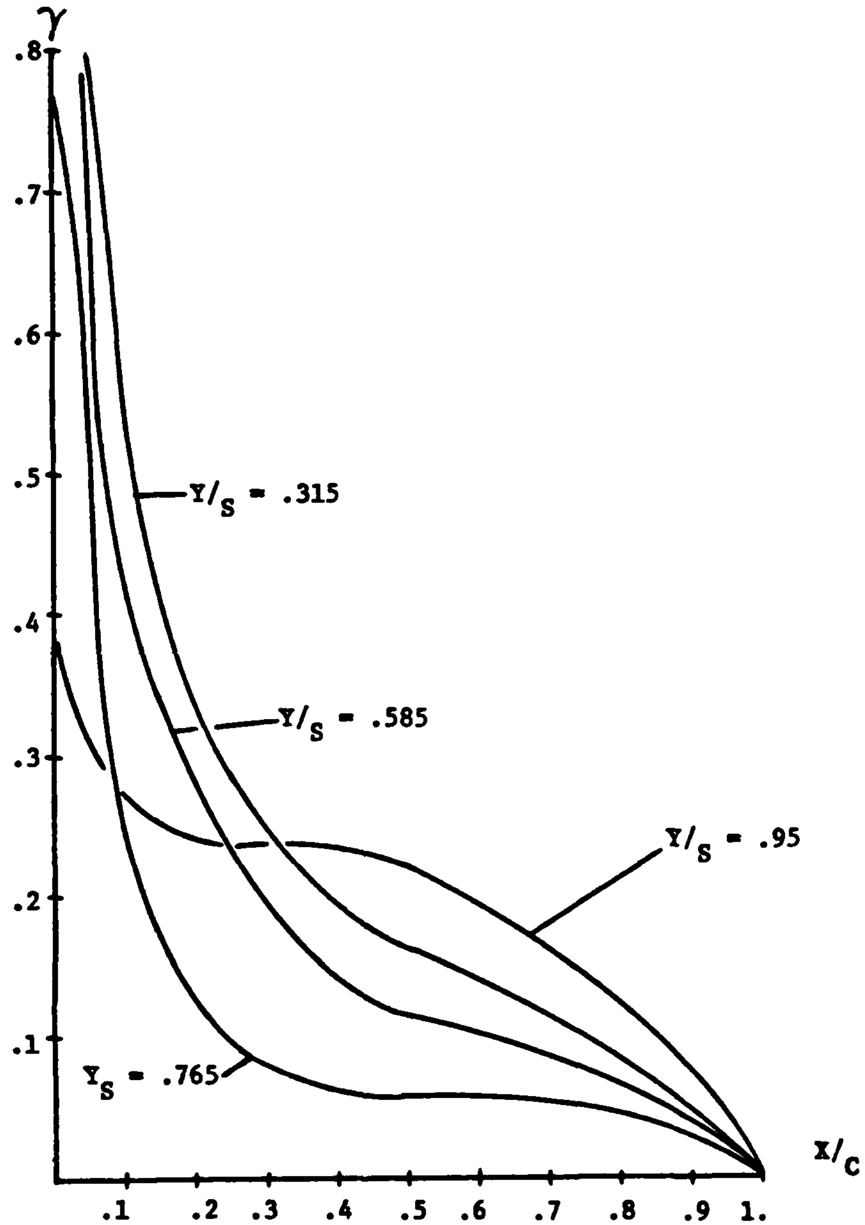

Figure 8. γ Strength Distribution at Selected Span Stations - Rectangular Wing; AR = 8, α = 5°

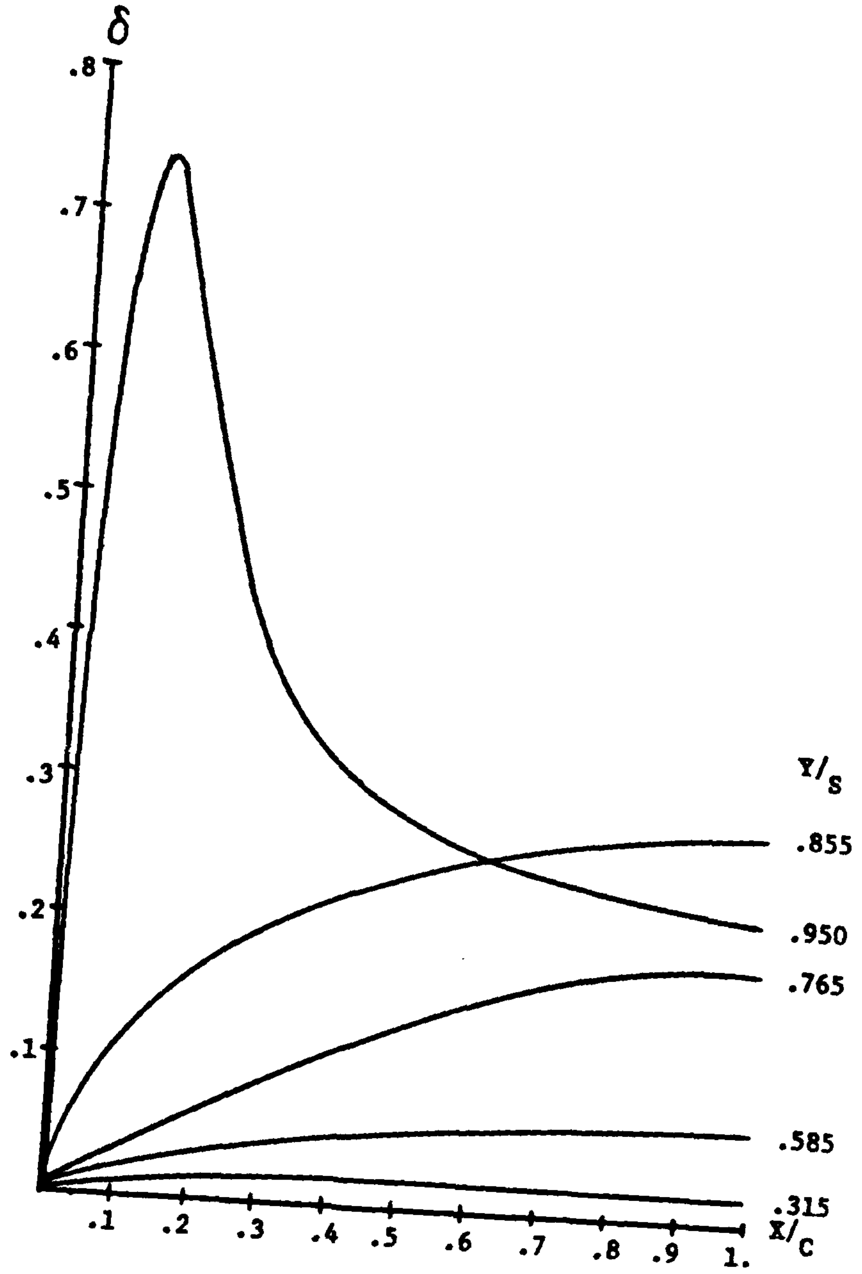

Figure 9. δ Strength Distribution at Selected Span Stations - Rectangular Wing; AR = 8, $\alpha = 5^{\circ}$

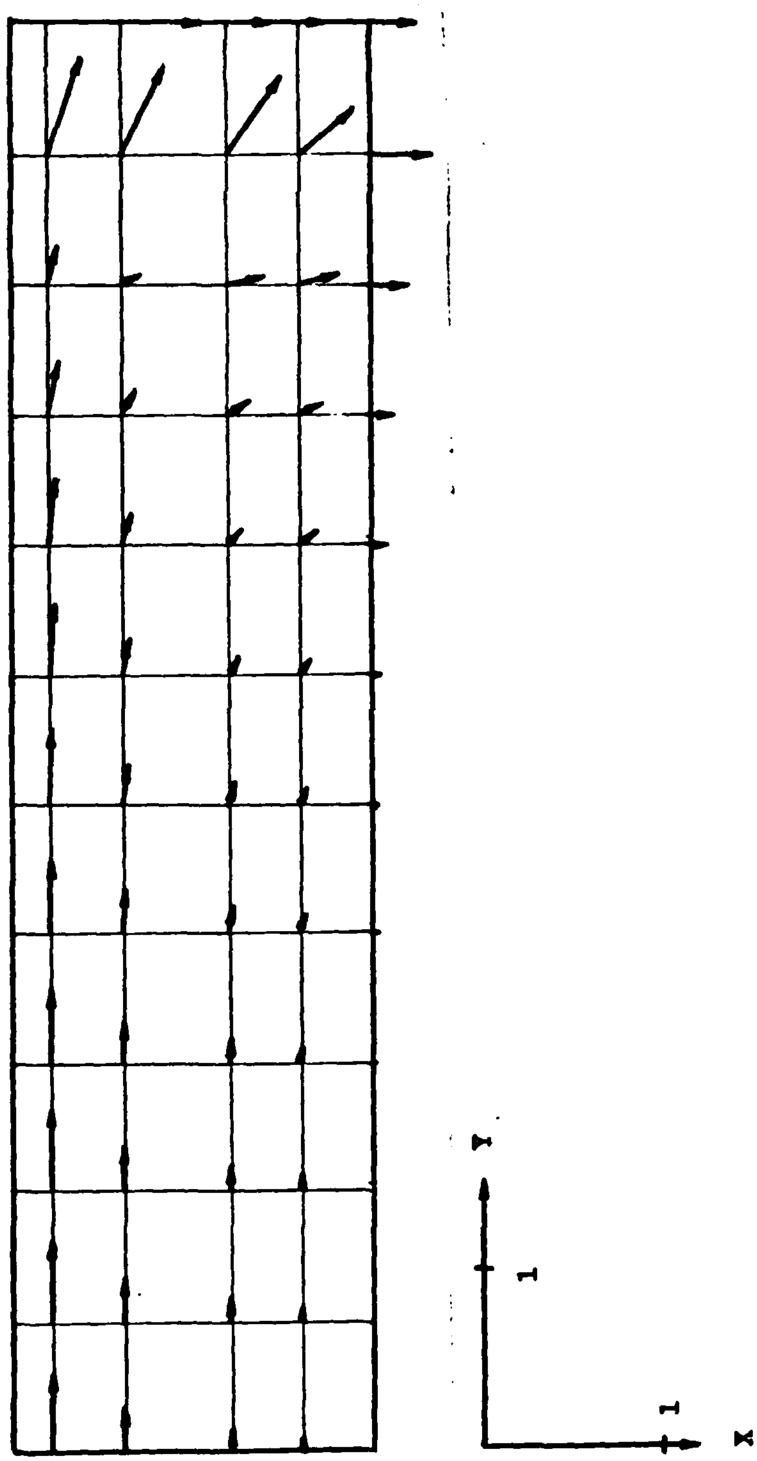

Figure 10. Relative Magnitude and Direction of the Vorticity Vector at Selected Corner Points - Rectangular Wing; AR = 8, α = 5°

is the condition of infinite slope (eq (3.5)). Also, the δ component is small near the wing root chord which illustrates the effect of planform symmetry. Both the δ and γ components exhibit unusual behavior in the tip region. Possible remedies for this problem are discussed in Section VI.

Swept Wing

The second case examined is a swept untapered wing; AR = 4.5, $\Lambda = 40^{\circ}$, $\alpha = 5^{\circ}$, and $M_{\infty} = .1$. The wing modeling is the same as the rectangular wing. Figure 11 shows the C_L distribution predicted by WING and by wind tunnel tests (Ref 4:92). Again, a spiking phenomena occurs. The predicted C_L curve is showing the proper curvature in the region inboard of the spike. Also, the total lift coefficients again show close agreement. Figure 12 shows the x center of pressure versus span station which again shifts aft in the region of spiking.

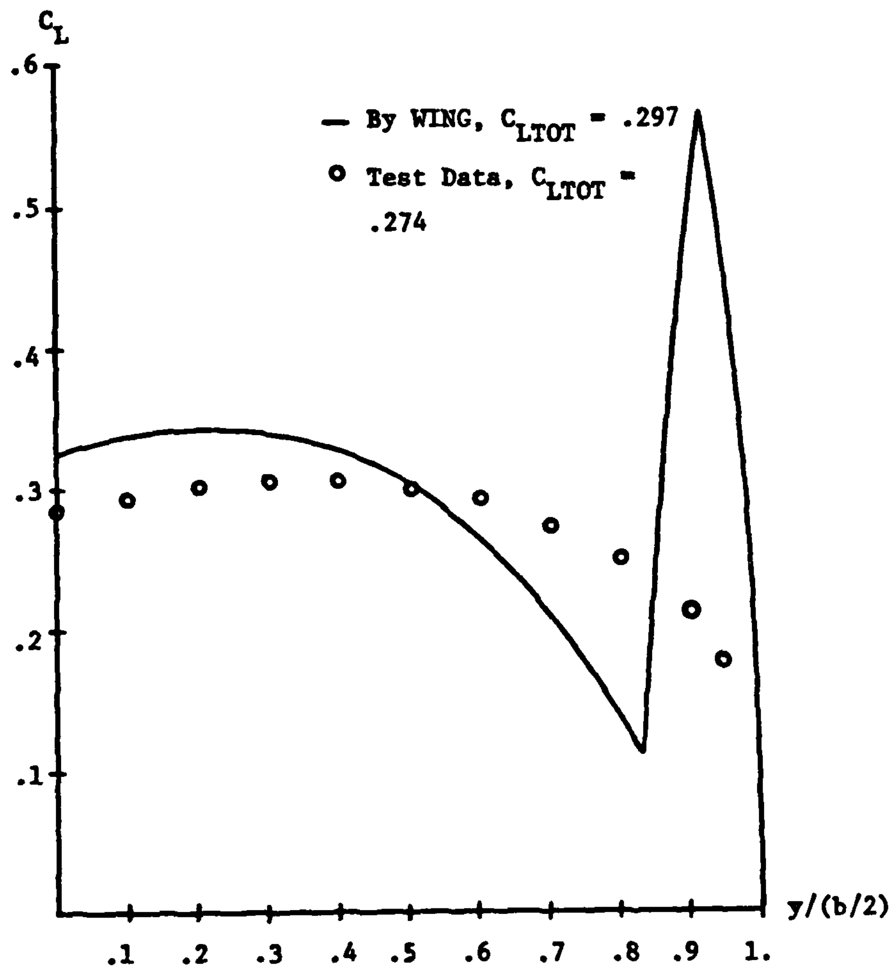

Figure 11. C_L Versus Span Station for a Sweptback Untapered Wing; AR = 4.5, $\Lambda = 40^{\circ}$, $\alpha = 5^{\circ}$

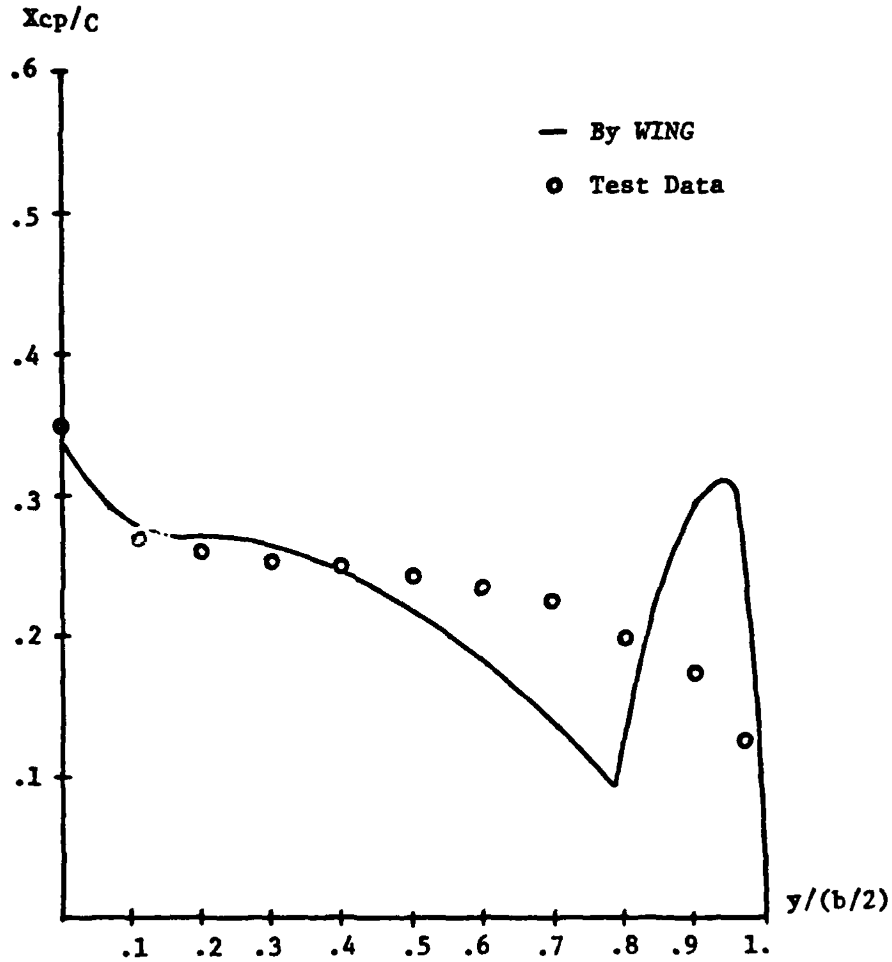

Figure 12. $X_{cp}/_C$ Versus Span Station for a Sweptback Untapered Wing; AR = 4.5, Λ = 40°, α = 5°

VI. Conclusions and Recommendations

Conclusions

A paneling technique which allows the vorticity vector to change direction has been demonstrated. The proper choice of control point (see Section V) will guarantee a solution free of numerical oscillations in the vorticity vector. This method has been implemented on a computer and introduces no new complexities to an experienced programmer. Existing mesh generators and other aerodynamic modules were incorporated into this technique (Ref 8). Computer run times are of the same order as programs incorporating "fixed direction vorticity" panels and no problems involving extensive "run times" were encountered.

The method gives good aerodynamic results near the centerline of the wing. However, the method will underpredict life as we move outboard. A gross overprediction of lift occurs in the region defined by the three most outboard span stations. Total lift coefficients as predicted by this technique agree well with existing solutions.

Recommendations

The following ideas are suggested for the improvement of the method and hopefully will lead to the elimination or minimization of the "spiking" problem.

a. A wake model should be incorporated into the program.

b. The flat plate panels should have a provision for leading edge suction.

c. Higher-order panels might be needed at the leading edge and tip. This would turn the vorticity vector faster and minimize the spiking. An elliptic vorticity distribution is suggested since many classic lift distributions are elliptic near the tip.

d. Change or modify the leading edge boundary condition. The "classical" tangency condition may be unappropriate for this kind of panel. Reference 9 suggests the Kutta condition be applied at the leading edge.

e. Interchange the role of γ and δ with respect to panel boundary continuity conditions. Perhaps the γ distribution enjoys too much continuity on the planform. This could be creating problems by forcing the γ distribution to undergo unusual "warping" in order to satisfy the planform boundary conditions.

Bibliography

1. Abbot, Ira H. and von Doenhoff, Albert E. Theory of Wing Sections. New York: Dover Publications Inc., 1959.

2. Ashley, Holt and Landahl, Marten. Aerodynamics of Wings and Bodies. Reading, Massachusetts: Addison-Wesley Publishing Company, Inc., 1965.

3. Hughes, William F. and Brighton, John A. Schaum's Outline Series: Fluid Dynamics. New York: McGraw-Hill Book Company, 1967.

4. Jones, Robert T. and Cohen, Doris. High Speed Wing Theory. Princeton: Princeton University Press, 1960.

5. Karamcheti, Krishnamurty. Principles of Ideal-Fluid Aerodynamics. New York: John Wiley and Sons, Inc., 1966.

6. Minter, E. A. and Rudnicki, A. R. Preliminary Structural Design Load Prediction Techniques for Military Aircraft, Volume II. AFFDL-TR-76-23. Wright-Patterson Air Force Base, Ohio: Air Force Flight Dynamics Laboratory, February 1976.

7. Pierce, B. O. A Short Table of Integrals. New York: Ginn and Company, 1929.

8. Sparks, John C. A Calculator Program for Analyzing Airloads on a Wing of Arbitrary Planform and Camber in Subsonic Flow. AFFDL-TR-77-136. Wright-Patterson Air Force Base, Ohio: Air Force Flight Dynamics Laboratory, January 1978.

9. Yen, A.; Mook, D. T.; and Nayfeh, A. H. "Nonlinear Continuous - Vorticity Aerodynamic Model." AIAA 18th Aerospace Sciences Meeting. Pasadena, California: American Institute of Aeronautics and Astronautics, January 1980.

APPENDIX A

Evaluation of the Panel Integrals

The methodology for evaluating the integrals $I_i^{L,\ T}$ of section 2 is presented here.

Consider the semi-infinite region shown in Figure 13. This strip is bound by the lines $y = y_o$, $y = y_1$, and the line segment connecting (x_o, y_o) to (x_1, y_1). The equation for the line segment is:

$$y = Mx + b \tag{A.1}$$

where

$$M = ((y_1 - y_o)/(x_1 - x_o)) \tag{A.2}$$

and

$$b = y_o - x_o M \tag{A.3}$$

Define the following improper integrals on the semi-infinite strip

$$F_i = \lim_{L \to \infty} \int_{y_o}^{y_1} \int_{(y - b)/M}^{L} f_i(x, y)dxdy \quad (i = 1, \text{---}5) \tag{A.4}$$

where

$$f_1(x, y) = x/(x^2 + y^2)^{3/2} \tag{A.5}$$

$$f_2(x, y) = x^2/(x^2 + y^2)^{3/2} \tag{A.6}$$

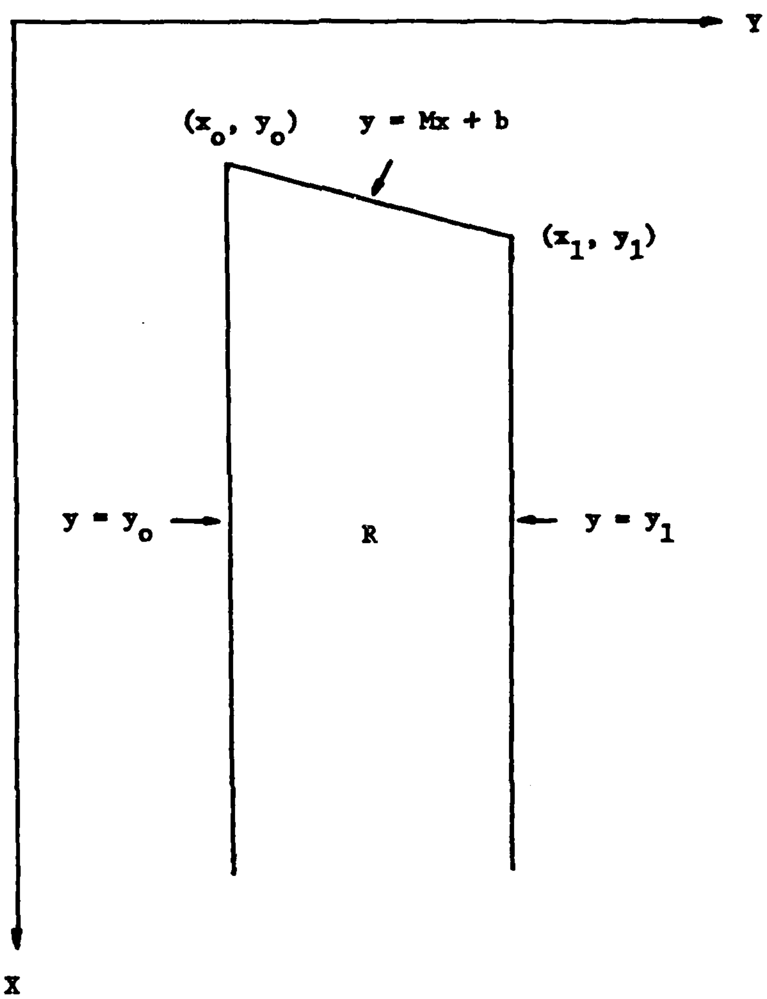

Figure 13. Semi-Infinite Strip R Used In Integral Evaluation

$$f_3(x, y) = xy/(x^2 + y^2)^{3/2} \tag{A.7}$$

$$f_4(x, y) = y/(x^2 + y^2)^{3/2} \tag{A.8}$$

and

$$f_5(x, y) = y^2/(x^2 + y^2)^{3/2} \tag{A.9}$$

To illustrate the methodology used in evaluating the integrals (A.4), consider

$$F_5 = \lim_{L\to\infty} \int_{y_o}^{y_1} \int_{(y-b)/M}^{L} [y^2/(x^2 + y^2)^{3/2}]dxdy \tag{A.10}$$

By Pierce's integral tables (Ref 7),

$$F_5 = \lim_{L\to\infty} \int_{y_o}^{y_1} [y^2x/(y^2\sqrt{x^2 + y^2}) \Big|_{(y-b)/M}^{L}]dy = \tag{A.11}$$

$$\lim_{L\to\infty} \int_{y_o}^{y_1} L\ dy/\sqrt{L^2 + y^2} - \int_{y_o}^{y_1} ((y - b)/M)dy/$$

$$\sqrt{((y - b)/M)^2 + y^2} \tag{A.12}$$

Substituting

$$z = (y - b)/M \tag{A.13}$$

and

$$dy = Mdz \tag{A.14}$$

into the rightmost integral of (A.12) yields:

$$F_5 = \lim_{L \to \infty} \int_{y_0}^{y_1} (L dy/\sqrt{L^2 + y^2}) - M \int_{x_0}^{x_1} z dz / \sqrt{z^2 + (Mz + b)^2} \tag{A.15}$$

The integrals (A.15) can be evaluated using Ref 7. One obtains, after some algebraic manipulation,

$$\begin{aligned} F_5 = \lim_{L \to \infty} & L \ln[(y_1 + \sqrt{y_1^2 + L^2})/(y_o + \sqrt{y_o^2 + L^2})] + \\ & [bM^2/(1 + M^2)^{3/2}] \ln[(\sqrt{x_1^2 + y_1^2} + ((x_1 + My_1)/ \\ & \sqrt{1 + M^2}))/(\sqrt{x_o^2 + y_o^2} + ((x_o + My_o)/\sqrt{1 + M^2}))] + \\ & [M/(1 + M^2)](\sqrt{x_o^2 + y_o^2} - \sqrt{x_1^2 + y_1^2}) \end{aligned} \tag{A.16}$$

recalling

$$y_o = Mx_o + b \tag{A.17}$$

and

$$y_1 = Mx_1 + b \tag{A.18}$$

Notice the limit

$$\lim_{L \to \infty} L \ln[(y_1 + \sqrt{y_1^2 + L^2})/(y_o + \sqrt{y_o^2 + L^2})] \tag{A.19}$$

is not a function of x_o or x_1 which leads to the following observation.

If the integral F_5 is evaluated on any other semi-infinite strip bounded by the lines $y = y_o$ and $y = y_1$, the limit (A.19) is invariant. Evaluating the integral F_5 using any other line segment connecting $y = y_o$ to $y = y_1$ and formulating the difference between this result and (A.16) leads to cancellation of the limit (A.19).

The other four integrals, F_1 through F_4, can be evaluated by a similar procedure using Ref 7. Each integral has a limit term given by:

$$(F_1) \quad \lim_{L\to\infty} \ln[(y_o + \sqrt{y_o^2 + L^2})/(y_1 + \sqrt{y_1^2 + L^2})] \qquad (A.20)$$

$$(F_2) \quad \lim_{L\to\infty}[y_1 \ln(\sqrt{L^2 + y_1^2} + L) - y_o \ln(\sqrt{L^2 + y_o^2} + L)] \qquad (A.21)$$

$$(F_3) \quad \lim_{L\to\infty}[\sqrt{L^2 + y_o^2} - \sqrt{L^2 + y_1^2}] \qquad (A.22)$$

$$(F_4) \quad \lim_{L\to\infty} \ln[(L + \sqrt{L^2 + y_o^2})/(L + \sqrt{L^2 + y_1^2})] \qquad (A.23)$$

These terms cancel upon the formulation of integral differences. Retaining the finite terms from each of the integral evaluations, we can define "Strip Functions" S_i for the functions f_i and the points (x_o, y_o) and (x_1, y_1) by

$$S_1[(x_o, y_o), (x_1, y_1)] = (M/\sqrt{1 + M^2})G \qquad (A.24)$$

$$S_2[(x_o, y_o), (x_1, y_1)] = y_o \ln[\sqrt{x_o^2 + y_o^2} + x_o] - y_1 \ln[\sqrt{x_1^2 + y_1^2} + x_1] + (M/(1 + M^2))H + [b/(1 + M^2)^{3/2}]G \tag{A.25}$$

$$S_3[(x_o, y_o), (x_1, y_1)] = (M^2/(1 + M^2))H + [bM/(1 + M^2)^{3/2}]G \tag{A.26}$$

$$S_4[(x_o, y_o), (x_1, y_1)] = \ln[(x_1 + \sqrt{x_1^2 + y_1^2})/(x_o + \sqrt{x_o^2 + y_o^2})] - G/\sqrt{1 + M^2} \tag{A.27}$$

$$S_5[(x_o, y_o), (x_1, y_1)] = [bM^2/(1 + M^2)^{3/2}]G - (M/(1 + M^2))H \tag{A.28}$$

where

$$G = \ln[(\sqrt{x_1^2 + y_1^2} + ((x_1 + My_1)/\sqrt{1 + M^2}))/(\sqrt{x_o^2 + y_o^2} + ((x_o + My_o)/\sqrt{1 + M^2}))] \tag{A.29}$$

and

$$H = \sqrt{x_1^2 + y_1^2} - \sqrt{x_o^2 + y_o^2} \tag{A.30}$$

Examining Figure 1, it is obvious that each of the panel integrals (2.30) through (2.34) can be obtained by formulating the difference of two corresponding strip functions. This leads to the following fundamental results:

$$I_i^L = S_i[(x_1, y_1), (x_3, y_2)] - S_i[(x_1, y_1), (x_4, y_2)] \qquad (A.31)$$

$$I_i^T = S_i[(x_1, y_1), (x_4, y_2)] - S_i[(x_2, y_1), (x_4, y_2)] \qquad (A.32)$$

where i ranges from 1 to 5.

APPENDIX B

SAMPLE OUTPUT

SPAN STATIONS

YLOC	Y/S
1	0.000
2	.250
3	.500
4	.750
5	1.000

SEMI-SPAN = 10.00

CHORD STATIONS

XLOC	X/C
1	0.000
2	.250
3	.500
4	.750
5	1.000

BREAK POINTS

LE	TE	YLOC
0.00	2.00	1
0.00	2.00	5

MACH NUMBER = .100

CY = .150
CX = .750

1

PANEL	X1	X2	X3	X4	Y1	Y2	XC	YC
1	0.00	.50	0.00	.50	0.00	2.50	.38	.38
2	.50	1.00	.50	1.00	0.00	2.50	.88	.38
3	1.00	1.50	1.00	1.50	0.00	2.50	1.38	.38
4	1.50	2.00	1.50	2.00	0.00	2.50	1.88	.38
5	0.00	.50	0.00	.50	2.50	5.00	.38	2.88
6	.50	1.00	.50	1.00	2.50	5.00	.88	2.88
7	1.00	1.50	1.00	1.50	2.50	5.00	1.38	2.88
8	1.50	2.00	1.50	2.00	2.50	5.00	1.88	2.88
9	0.00	.50	0.00	.50	5.00	7.50	.38	5.38
10	.50	1.00	.50	1.00	5.00	7.50	.88	5.38
11	1.00	1.50	1.00	1.50	5.00	7.50	1.38	5.38
12	1.50	2.00	1.50	2.00	5.00	7.50	1.88	5.38
13	0.00	.50	0.00	.50	7.50	10.00	.38	7.88
14	.50	1.00	.50	1.00	7.50	10.00	.88	7.88
15	1.00	1.50	1.00	1.50	7.50	10.00	1.38	7.88
16	1.50	2.00	1.50	2.00	7.50	10.00	1.88	7.88

ALPHA = 5.00

PRESSURE AND SINGULARITY STRENGTH DISTRIBUTION

XP = .750
YP = .150

LINEAR

Y/S = .038

X/C	CAMBER	DEL	GAM	CPU	CPL	DELCP
.188	0.00	.00215	.41582	-.46623	.36541	.83164
.438	0.00	.00357	.19382	-.21074	.17691	.38765
.688	0.00	.00452	.12030	-.13151	.10910	.24061
.938	0.00	.00484	.02656	-.03436	.01877	.05313

Y/S = .288

X/C	CAMBER	DEL	GAM	CPU	CPL	DELCP
.188	0.00	.00618	.38896	-.43403	.34389	.77792
.438	0.00	.01164	.17955	-.19518	.16392	.35910
.688	0.00	.01598	.11280	-.12363	.10197	.22560
.938	0.00	.01862	.02556	-.03343	.01770	.05113

Y/S = .538

X/C	CAMBER	DEL	GAM	CPU	CPL	DELCP
.188	0.00	.05032	.32041	-.35407	.28675	.64082
.438	0.00	.07086	.14394	-.15793	.12994	.28787
.688	0.00	.08090	.09528	-.10678	.08378	.19057
.938	0.00	.08360	.01946	-.02892	.01000	.03892

Y/S = .788

X/C	CAMBER	DEL	GAM	CPU	CPL	DELCP
.188	0.00	.21639	.33951	-.38736	.29166	.67902
.438	0.00	.17284	.21860	-.24551	.19169	.43720
.688	0.00	.17610	.15057	-.17154	.12959	.30113
.938	0.00	.18774	.03844	-.05524	.02165	.07689

LOADS SUMMARY

YS	X-CP	CHORD	CL	CM
.04	.28	2.00	.48	.134
.29	.28	2.00	.45	.126
.54	.27	2.00	.37	.102
.79	.34	2.00	.45	.152

APPENDIX C

PROGRAM LISTING

```
      PROGRAM WING(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C                    PROGRAM WING CALCULATES THE AIRLOADS ON A
C                    WING OF ARBITRARY PLANFORM AND CAMBER IN
C                    SUBSONIC FLOW. A TRIANGULAR PANEL HAVING
C                    A BILINEAR VORTICITY DISTRIBUTION IS USED
C                    TO PREDICT THE PRESSURE FIELD. COMPRESS-
C                    IBILITY EFFECTS ARE ACCOUNTED FOR THROUGH
C                    THE PRANDTL-GLAUERT TRANSFORMATION.
C
      COMMON/BLOCKA/X(60,4),Y(60,2),XC(60),YC(60),E(60,2),CY,CX
      COMMON/BLOCKB/NS,SSPN,S(15),NC,C(10),NB,NL(10),B(10,2),MACH
      COMMON/BLOCKC/A(130,130),SG(130),CBR(60),SUM(60),ALPHA,NA
      COMMON/BLOCKD/IL(5),IT(5)
      COMMON/BLOCKE/SPF(5,3)
      REAL MACH
      REAL IL,IT
C
C                    BEGIN INPUT SEQUENCE FOR GEOMETRIC DATA.
C                    ALL DATA IS READ USING FREE FORMAT.
C                    READ SPAN DATA.
      READ(5,*) SSPN,NS,(S(I),I=1,NS)
      WRITE(6,100)
  100 FORMAT(////29X,*SPAN STATIONS*/26X,*YLOC*,11X,*Y/S*)
      DO 10 I=1,NS
   10 WRITE(6,110) I,S(I)
  110 FORMAT(26X,I3,10X,F6.3)
      WRITE(6,120) SSPN
  120 FORMAT(/27X,*SEMI-SPAN =*,F7.2)
C                    READ CHORD DATA.
      READ(5,*) NC,(C(I),I=1,NC)
      WRITE(6,130)
  130 FORMAT(//28X,*CHORD STATIONS*/26X,*XLOC*,11X,*X/C*)
      DO 20 I=1,NC
   20 WRITE(6,140) I,C(I)
  140 FORMAT(26X,I3,10X,F6.3)
C                    READ BREAK POINT DATA.
      READ(5,*) NB,((NL(I),B(I,1),B(I,2)),I=1,NB)
      WRITE(6,150)
  150 FORMAT(//29X,*BREAK POINTS*/22X,*LE*,10X,*TE*,10X,*YLOC*)
      DO 30 I=1,NB
   30 WRITE(6,160) B(I,1),B(I,2),NL(I)
  160 FORMAT(19X,F7.2,5X,F7.2,7X,I3)
C                    READ CONTROL POINT LOCATIONS AND MACH
      READ(5,*) CY,CX,MACH
      WRITE(6,170) MACH,CY,CX
  170 FORMAT(//27X,*MACH NUMBER =*,F6.3//31X,*CY =*,F6.3/31X,
     1*CX =*,F6.3)
C                    READ THE NUMBER OF LOADING CASES TO BE
C                    EXAMINED. THE ANGLE OF ATTACK AND CAMBER
C                    DISTRIBUTION FOR EACH CASE WILL BE READ
C                    LATER IN THE PROGRAM. THIS IS DONE TO
C                    PRESERVE STORAGE SPACE.
      READ(5,*) NA
      WRITE(6,*)NA
C                    CALCULATE M AND N. M IS THE NUMBER OF
C                    CHORDWISE PANELS AND N IS THE NUMBER
```

```
      C                    OF SPANWISE PANELS
            M=NC-1
            N=NS-1
      C                    CALCULATE NUMBER OF PANELS NP.
            NP=M*N
      C                    CALCULATE NUMBER OF UNKNOWN SINGULARITIES.
            NP2=2*NP+M
      C                    CALL MESH GENERATOR
            CALL MESH(NP)
            CALL AERO(NP,M,N,NP2)
            CALL INVRT(NP2)
            CALL LOADS(NP,M,N,NP2)
            END
```

```
      SUBROUTINE MESH(NP)
C                   SUBROUTINE TO CALCULATE PANEL CORNER POIN
C                   TS AND CONTROL POINTS.
      COMMON/BLOCKA/X(60,4),Y(60,2),XC(60),YC(60),E(60,2),CY,CX
      COMMON/BLOCKB/NS,SSPN,S(15),NC,C(10),NB,NL(10),B(10,2),MACH
      NP=1
      NB1=NB-1
      NC1=NC-1
      DO 40 K=1,NB1
      DO 10 I=1,NC
      E(I,1)=B(K,1)+C(I)*(B(K,2)-B(K,1))
   10 E(I,2)=B(K+1,1)+C(I)*(B(K+1,2)-B(K+1,1))-E(I,1)
      IC1=NL(K)
      IC2=NL(K+1)
      IC3=IC2-1
      DO 30 J=IC1,IC3
      D2=S(IC2)-S(IC1)
      D3=S(IC1)
      DO 20 I=1,NC1
      X(NP,1)=E(I,1)+E(I,2)*(S(J)-D3)/D2
      X(NP,2)=E(I+1,1)+E(I+1,2)*(S(J)-D3)/D2
      X(NP,3)=E(I,1)+E(I,2)*(S(J+1)-D3)/D2
      X(NP,4)=E(I+1,1)+E(I+1,2)*(S(J+1)-D3)/D2
      Y(NP,1)=S(J)*SSPN
      Y(NP,2)=S(J+1)*SSPN
      NP=NP+1
   20 CONTINUE
   30 CONTINUE
   40 CONTINUE
C                   CALCULATE CONTROL POINTS.
      DO 50 I=1,NP
      YC(I)=(1-CY)*Y(I,1)+CY*Y(I,2)
   50 XC(I)=CY*X(I,2)+(1-CY)*X(I,1)+CX*(CY*(X(I,4)-X(I,3))+(1-CY)*
     1(X(I,2)-X(I,1)))
C                   PRINT A TABLE CONSISTING OF THE PANEL NO.
C                   AND THE CORRESPONDING CORNER AND CONTROL
C                   POINTS.
      WRITE(6,100)
  100 FORMAT(////5X,*PANEL*,7X,*X1*,9X,*X2*,8X,*X3*,9X,*X4*,9X,*Y1*,
     19X,*Y2*,9X,*XC*,8X,*YC*)
      DO 60 I=1,NP
      WRITE(6,101) I,X(I,1),X(I,2),X(I,3),X(I,4),Y(I,1),Y(I,2),XC(I),
     1YC(I)
  101 FORMAT(6X,I2,8(4X,F7.2))
   60 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INT(X1,X2,X3,X4,Y1,Y2)
C              THIS SUBROUTINE EVALUATES THE PANEL INTE-
C              GRALS. IT ALSO FUNCTIONS AS THE EXECUTIVE
C              CONTROL ROUTINE FOR SUBROUTINE STRIP.
      COMMON/BLOCKD/IL(5),IT(5)
      COMMON/BLOCKE/SPF(5,3)
      REAL IL,IT
C              CALCULATE STRIP FUNCTIONS FOR THE PANEL
C              LEADING EDGE WITH CORNER POINTS (X1,Y1)
C              AND (X3,Y2).
      I=1
      CALL STRIP(X1,Y1,X3,Y2,I)
C              CALCULATE STRIP FUNCTIONS FOR THE PANEL
C              MAIN DIAGONAL WITH CORNER POINTS (X1,Y1)
C              AND(X4,Y2).
      I=2
      CALL STRIP(X1,Y1,X4,Y2,I)
C              CALCULATE STRIP FUNCTIONS FOR THE PANEL
C              TRAILING EDGE WITH CORNER POINTS (X2,Y1)
C              AND (X4,Y2).
      I=3
      CALL STRIP(X2,Y1,X4,Y2,I)
C              EVALUATE PANEL INTEGRALS.
      DO 10 I=1,5
      IL(I)=SPF(I,1)-SPF(I,2)
      IT(I)=SPF(I,2)-SPF(I,3)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STRIP(X0,Y0,X1,Y1,I)
C                SUBROUTINE TO CALCULATE STRIP FUNCTIONS.
C                THERE ARE 15 STRIP FUNCTION CALCULATIONS
C                PER QUADRILATERAL PANEL.
      COMMON/BLOCKE/SPF(5,3)
      REAL MO
C                USE STRIP FUNCTION EQUATIONS FOR AN IN-
C                FINITE SLOPE IF X1=X0.
      IF(ABS(X1-X0).LT.1E-8) GO TO 100
      MO=(Y1-Y0)/(X1-X0)
      BO=Y0-X0*MO
      DUM1=SQRT(X0**2+Y0**2)
      DUM2=SQRT(X1**2+Y1**2)
      DUM3=SQRT(1+MO**2)
      DUM4=(DUM1+(X0+MO*Y0)/DUM3)/(DUM2+(X1+MO*Y1)/DUM3)
      DUM4=ABS(DUM4)
      DUM4=ALOG(DUM4)
      DUM5=ABS(DUM1+Y0)
      DUM5=ALOG(DUM5)
      DUM6=ABS(DUM2+Y1)
      DUM6=ALOG(DUM6)
      SPF(1,I)=-MO*DUM4/DUM3
      SPF(2,I)=Y0*DUM5-Y1*DUM6-MO*(DUM1-DUM2)/DUM3**2-BO*DUM4/DUM3**3
      SPF(3,I)=MO**2*(DUM2-DUM1)/DUM3**2-BO*MO*DUM4/DUM3**3
      SPF(4,I)=DUM4/DUM3+DUM6-DUM5
      SPF(5,I)=-BO*MO**2*DUM4/DUM3**3+MO*(DUM1-DUM2)/DUM3**2
      RETURN
  100 CONTINUE
      DUM1=SQRT(X0**2+Y0**2)
      DUM2=SQRT(X0**2+Y1**2)
      DUM3=ABS((DUM1+Y0)/(DUM2+Y1))
      SPF(1,I)=-ALOG(DUM3)
      DUM3=ABS(DUM1+X0)
      DUM3=ALOG(DUM3)
      DUM4=ABS(DUM2+X0)
      DUM4=ALOG(DUM4)
      SPF(2,I)=Y0*DUM3-Y1*DUM4
      SPF(3,I)=DUM2-DUM1
      SPF(4,I)=DUM4-DUM3
      SPF(5,I)=-X0*SPF(1,I)
      RETURN
      END
```

```
      SUBROUTINE INVERT(NP2)
C                SUBROUTINE TO INVERT THE AERODYNAMIC IN-
C                FLUENCE COEFFICIENT ARRAY USING GAUSSIAN
C                ELIMINATION.
      COMMON/BLOCKC/A(130,130),SG(130),CBR(60),SUP(60),ALPHA,NA
      DO 20 I=1,NP2
      PIVOT=A(I,I)
      A(I,I)=1.
      DO 10 L=1,NP2
   10 A(I,L)=A(I,L)/PIVOT
      DO 20 M=1,NP2
      IF(M.EQ.I) GO TO 20
      TT=A(M,I)
      A(M,I)=0.0
      DO 15 L=1,NP2
   15 A(M,L)=A(M,L)-A(I,L)*TT
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AERO(NP,M,N,NP2)
C
C
C                    THIS SUBROUTINE CALCULATES THE AERODYNA-
C                    MIC INFLUENCE COEFFICIENTS ASSUMING A
C                    VORTICITY DISTRIBUTION OF (-CX+EY+F)I+
C                    (A+BX+CY)J. THERE ARE 2(NP) UNKNOWNS
C                    AFTER THE BOUNDARY CONDITIONS HAVE BEEN
C                    APPLIED.
C
C
      COMMON/BLOCKA/X(60,4),Y(60,2),XC(60),YC(60),E(60,2),CY,CX
      COMMON/BLOCKB/NS,SSPN,S(15),NC,C(10),NB,NL(10),R(10,2),MACH
      COMMON/BLOCKC/A(130,130),SG(130),CNR(60),SUM(60),ALPHA,NA
      COMMON/BLOCKD/IL(5),IT(5)
      REAL MACH,IL,IT,KL,KT
      INTEGER D1,D2,D3,D4,G1,G2,G3,G4
      PI=3.141592
      AMCH=SQRT(1-MACH**2)
      MC=0
C
C                    ZERO OUT THE AERODYNAMIC INFLUENCE
C                    COEFFICIENT AND CONTINUITY CONDITION
C                    ARRAY A(J,I).
C
      DO 10 I=1,NP2
      DO 5 J=1,NP2
      A(J,I)=0.
    5 CONTINUE
   10 CONTINUE
C
C                    START GRAND CONTROL POINT EQUATION LOOP
C
      DO 400 J=1,NP
C                    RETRIEVE CONTROL POINTS AT PANEL J
      XCJ=XC(J)
      YCJ=YC(J)
C
C                    CALCULATE THE INDUCED VELOCITY AT CONTROL
C                    POINT J DUE TO PANEL I.
C
      DO 370 I=1,NP
C
C                    APPLY A LINEAR TRANSFORMATION TO THE CO-
C                    ORDINATES OF PANEL I WHERE THE NEW ORIGIN
C                    IS (XCJ,YCJ).
C
      X1=X(I,1)-XCJ
      X2=X(I,2)-XCJ
      X3=X(I,3)-XCJ
      X4=X(I,4)-XCJ
C
C                    CALCULATE THE 8 SINGULARITY NUMBERS AS A
C                    FUNCTION OF PANEL NUMBER.
C
C                    DELTA1
      D1=I-1
```

```
C                    DELTA2
      D2=I
C                    DELTA3
      D3=I-1+M
C                    DELTA4
      D4=I+M
C                    KAPPA1
      K1=NP+M+I
C                    KAPPA2
      K2=NP+M+I+1
C                    KAPPA3
      K3=NP+2*M+I
C                    KAPPA4
      K4=NP+2*M+I+1
C
C                    APPLY THE PRANDTL-GLAUERT TRANSFORMATION
C
      X1=X1/APCH
      X2=X2/APCH
      X3=X3/APCH
      X4=X4/APCH
C                    CALCULATE LEADING EDGE PANEL CHECK PARAM.
C                    NOTICE THE USE OF INTEGER ARITHMETIC.
      NE=I-(I/M)*M-1
C                    CALCULATE TRAILING EDGE PANEL CHECK PARAM.
      NS=I-(I/M)*M
C
C                    THIS LOOP COMPUTES THE INDUCED VELOCITY
C                    AT CONTROL POINT J DUE TO PANEL I (K=1)
C                    AND THE IMAGE PANEL OF I (K=2).
C
      DO 300 K=1,2
      DUM=(-1.)**K
      Y1=Y(I,1)+YCJ*DUM
      Y2=Y(I,2)+YCJ*DUM
C
C                    EVALUATE THE TEN PANEL INTEGRALS.
C
      CALL INT(X1,X2,X3,X4,Y1,Y2)
C
      KL=(2*(Y1-Y2)*IL(3)-(X1-X3)*IL(5)-(Y1*X3-X1*Y2)*IL(4))/(Y1-Y2)
      KT=(2*(Y1-Y2)*IT(3)-(X2-X4)*IT(5)-(Y1*X4-X2*Y2)*IT(4))/(Y1-Y2)
C                    CHECK IF PANEL I IS A WING TIP PANEL.
      IF(I.GT.(NP-M)) GO TO 200
C
C                    PANEL EQUATIONS IN THIS SECTION OF THE
C                    CODE ARE FOR ROOT CHORD AND INTERIOR PANELS.
C
      IF(NE.NE.0) GO TO 20
C                    APPLY FLOW TANGENCY CONDITION AT THE L.E.
      Y3=Y(I+M,2)+YCJ*DUM
      X5=(X(I+M,3)-XCJ)/APCH
C
      A(J,K1)=A(J,K1)+(Y2*IL(4)-IL(5))*(X3-X1)/(Y1-Y2)/(Y2-Y1)/4/PI
C
      A(J,K3)=A(J,K3)+(IL(5)-Y1*IL(4))*(X5-X3)/(Y1-Y2)/(Y3-Y2)/4/PI
      GO TO 30
```

```
   20 CONTINUE
C
      A(J,C1)=A(J,C1)+(Y2*IL(4)-IL(5))/(Y1-Y2)/4/PI
C
      A(J,C3)=A(J,C3)+(IL(5)-Y1*IL(4))/(Y1-Y2)/4/PI
C
   30 CONTINUE
C
      A(J,C1)=A(J,C1)+((X4*Y1-Y2*X2)*IT(1)-(X2-X1)*Y2*IL(1)-(Y1-Y2)
     1*IT(2)+(X2-X1)*KL+(X2-X4)*KT)/(X2-X1)/(Y1-Y2)/4/PI
C
      A(J,C2)=A(J,C2)+(Y2*IT(4)-IT(5))/(Y1-Y2)/4/PI
C
      A(J,C3)=A(J,C3)+((Y1*X4-X1*Y2)*IL(1)+(Y2-Y1)*IL(2)+(X1-X4)*KL)
     1/(Y2-Y1)/(X3-X4)/4/PI
C
      A(J,C4)=A(J,C4)+(IT(5)-Y1*IT(4))/(Y1-Y2)/4/PI
C
      IF(NS.EQ.NC) GO TO 360
C
      A(J,C2)=A(J,C2)+((Y2*X1-X4*Y1)*IT(1)+(Y1-Y2)*IT(2)+(X4-X1)*KT)
     1/(Y1-Y2)/(X2-X1)/4/PI
C
      A(J,C4)=A(J,C4)+((X1*Y2-Y1*X3)*IL(1)-(X3-X4)*Y1*IT(1)-(Y2-Y1)
     1*IL(2)+(X3-X1)*KL+(X3-X4)*KT)/(Y2-Y1)/(X3-X4)/4/PI
C
      GO TO 360
  200 CONTINUE
C
C                  PANEL EQUATIONS IN THIS SECTION OF THE
C                  CODE ARE FOR TIP CHORD PANELS
C
      IF(NE.NE.NC) GO TO 220
C                  APPLY FLOW TANGENCY CONDITION AT THE L.E.
C                  NOTE - GAMMA3 = 0. IMPLIES DELTA3 = 0.
C                  AT THE L.E.
C
      A(J,C1)=A(J,C1)+(Y2*IL(4)-IL(5))*(X3-X1)/(Y1-Y2)/(Y2-Y1)/4/PI
      GO TO 230
  220 CONTINUE
C
      A(J,C1)=A(J,C1)+(Y2*IL(4)-IL(5))/(Y1-Y2)/4/PI
C
      A(J,C3)=A(J,C3)+(IL(5)-Y1*IL(4))/(Y1-Y2)/4/PI
C
  230 CONTINUE
C
      A(J,C2)=A(J,C2)+(Y2*IT(4)-IT(5))/(Y1-Y2)/4/PI
C
      A(J,C4)=A(J,C4)+(IT(5)-Y1*IT(4))/(Y1-Y2)/4/PI
C
      A(J,C1)=A(J,C1)+((X4*Y1-Y2*X2)*IT(1)-(X2-X1)*Y2*IL(1)-(Y1-Y2)
     1*IT(2)+(X2-X1)*KL+(X2-X4)*KT)/(X2-X1)/(Y1-Y2)/4/PI
C
      IF(NS.EQ.NC) GO TO 360
C
      A(J,C2)=A(J,C2)+((Y2*X1-X4*Y1)*IT(1)+(Y1-Y2)*IT(2)+(X4-X1)*KT)
```

```
               1/(Y1-Y2)/(X2-X1)/4/PI
      C
        360    CONTINUE
        370    CONTINUE
        400    CONTINUE
      C
      C                         THIS SECTION OF SUBROUTINE AERO FORMULATES
      C                         THE NP DELTA SPECIAL CONTINUITY CONDITIONS
      C                         AT THE LOWER RIGHT HAND PANEL CORNER.
      C
               DO 600 I=1,NP
               INP=I+NP
               X1=X(I,1)/ARCH
               X2=X(I,2)/ARCH
               X3=X(I,3)/ARCH
               X4=X(I,4)/ARCH
               Y1=Y(I,1)
               Y2=Y(I,2)
      C
               D3=I-1
               D4=I
               C1=NP+I
               G3=NP+I+M
               G4=NP+I+M+1
      C                         CALCULATE TE AND LE CHECK PARAMETERS.
               N8=I-((I/M)*M-1
               N9=I-(I/M)*M
      C                         CHECK IF PANEL I IS A WING TIP PANEL.
               IF(I.GT.(NP-M)) GO TO 500
      C
      C                         PANEL CONTINUITY EQUATIONS IN THIS SECTION
      C                         OF THE CODE ARE FOR INTERIOR AND ROOT
      C                         CHORD PANELS.
      C
               IF(N8.NE.MC) GO TO 410
               Y3=Y(I+M,2)
               X5=X(I+M,3)/ARCH
               A(INP,G3)=-(X5-X3)/(Y3-Y2)
               GO TO 420
        410    CONTINUE
               A(INP,D3)=-1.
        420    CONTINUE
               A(INP,D4)=1.
               A(INP,G3)=A(INP,G3)-(X1-X4)/(Y2-Y1)
               A(INP,C1)=(X3-X4)/(Y2-Y1)
               IF(N9.EQ.MC) GO TO 590
               A(INP,G4)=-(X3-X1)/(Y2-Y1)
               GO TO 590
      C
      C                         PANEL CONTINUITY EQUATIONS IN THIS SECTION
      C                         OF THE CODE ARE FOR TIP CHORD PANELS.
      C
        500    CONTINUE
               IF(N8.EQ.MC) GO TO 510
               A(INP,D3)=-1.
        510    CONTINUE
               A(INP,D4)=1.
```

```
      A(INP,G1)=(X3-X4)/(Y2-Y1)
  590 CONTINUE
  600 CONTINUE
C
C                       THIS SECTION OF SUBROUTINE AERO FORMULATES
C                       M ADDITIONAL DELTA CONTINUITY CONDITIONS
C                       ON THE ROOT CHORD PANELS.
C
      DO 700 I=1,M
      INP=2*NP+I
      J=I
      X1=X(J,1)/AMCH
      X2=X(J,2)/AMCH
      X3=X(J,3)/AMCH
      X4=X(J,4)/AMCH
      Y1=Y(J,1)
      Y2=Y(J,2)
C
      D1=J-1
      D2=J
      G1=NP+M+J
      G2=NP+M+J+1
      G4=NP+2*M+J+1
C                       CALCULATE TE AND LE CHECK PARAMETERS.
      N8=J-(J/M)*M-1
      N9=J-(J/M)*M
      IF(N8.NE.MC) GO TO 710
      A(INP,G1)=-(X3-X1)/(Y2-Y1)
      GO TO 720
  710 CONTINUE
      A(INP,D1)=-1.
  720 CONTINUE
      A(INP,D2)=1.
      A(INP,G1)=A(INP,G1)+(X2-X4)/(Y1-Y2)
      IF(N9.EQ.MC) GO TO 700
      A(INP,G4)=-(X2-Y1)/(Y1-Y2)
      A(INP,G2)=(X4-X1)/(Y1-Y2)
  700 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE LOADS(NP,M,N,NP2)
C                   SUBROUTINE TO CALCULATE PRESSURE DISTRI-
C                   BUTIONS AND AERODYNAMIC COEFFICIENTS.
      COMMON/BLOCKA/X(60,4),Y(60,2),XC(60),YC(60),E(60,2),CY,CX
      COMMON/BLOCKB/NS,SSPN,S(15),NC,C(10),NB,NL(10),B(10,2),MACH
      COMMON/BLOCKC/A(130,130),SG(130),CBR(60),SUM(60),ALPHA,NA
      DIMENSION DEL(60),GAM(60),CPU(60),CPL(60),CLL(60),CM(60)
      INTEGER D1,D2,D3,D4,G1,G2,G3,G4
      REAL MACH
      G=1.4
      AMCH=SQRT(1-MACH**2)
      FC=0
C                   SET CAMBER SLOPES EQUAL TO ZERO.
      DO 10 I=1,NP
      CBR(I)=0.
   10 CONTINUE
C                   READ NA SETS OF ANGLE OF ATTACK AND
C                   CAMBER SLOPE DISTRIBUTION DATA.
      DO 800 L=1,NA
      READ(5,*) ALPHA,NCCG,NPF,CY,CX
C
C                   NCCG IS THE CAMBER CHANGE PARAMETER.
C                   ENTER 0 TO READ A NEW CAMBER SLOPE DISTRI-
C                   BUTION OR ENTER 1 TO RETAIN THE PREVIOUS
C                   DISTRIBUTION.
C
C                   NPF IS THE PRESSURE OPTION PARAMETER.
C                   ENTER 0 TO USE THE EXACT ISENTROPIC
C                   EXPRESSION OR ENTER 1 TO USE THE LINEAR-
C                   IZED FORM.
C
C                   COMPUTE PRESSURES AT PANEL LOCATION CX
C                   AND CY. NOTE - PRESSURES MAY BE COMPUTED
C                   AT POINTS OTHER THAN THE CONTROL POINTS.
C
C                   CALCULATE PRESSURE EVALUATION POINTS.
      DO 35 I=1,NP
      YC(I)=(1-CY)*Y(I,1)+CY*Y(I,2)
   35 XC(I)=CY*X(I,2)+(1-CY)*X(I,1)+CX*((CY*(X(I,4)-X(I,3))+(1-CY)*
     *(X(I,2)-X(I,1))))
      IF(NCCG.EQ.1) GO TO 20
      READ(5,*) (CBR(I),I=1,NP)
   20 CONTINUE
C                   FORMULATE LINEARIZED FORM OF THE FLOW
C                   TANGENCY BOUNDARY CONDITION. SLP(I)
C                   REPRESENTS THE NORMALIZED Z COMPONENT OF
C                   VELOCITY FOR PANEL I.
      DO 30 I=1,NP
      SLP(I)=(CBR(I)-ALPHA)/57.2957795
   30 CONTINUE
C                   PREMULTIPLY SLP ARRAY BY A TO OBTAIN THE
C                   UNKNOWN SINGULARITY STRENGTHS.
      DO 50 J=1,NP2
      SUM=0.
      DO 40 I=1,NP
      SUM=SUM+A(J,I)*SLP(I)
   40 CONTINUE
```

```
      S(I,J)=ZERO
   50 CONTINUE
      LT(I)=0.
C
C                        START LOOP FOR COMPUTING THE SINGULARITY
C                        STRENGTH AT THE CONTROL POINT.
C
      DO 600 I=1,NP
C                        RETRIEVE CONTROL POINTS AND PANEL CORNER
C                        POINTS. APPLY PRANDTL-GLAUERT TRANSFORM.
      XI=XC(I)/AMCH
      YI=YC(I)
      X1=X(I,1)/AMCH
      X2=X(I,2)/AMCH
      X3=X(I,3)/AMCH
      X4=X(I,4)/AMCH
      Y1=Y(I,1)
      Y2=Y(I,2)
C                        CALCULATE THE 8 SINGULARITY NUMBERS.
      D1=I-1
      D2=I
      D3=I-1+M
      D4=I+M
      G1=NP+M+I
      G2=NP+M+I+1
      G3=NP+2*M+I
      G4=NP+2*M+I+1
C                        CALCULATE TE AND LE CHECK PARAMETERS.
      N8=I-((I/M)*M-1
      N9=I-(I/M)*M
C                        INITIALLY SET ALL SINGULARITIES TO 0.
      GAM1=0.
      GAM2=0.
      GAM3=0.
      GAM4=0.
      DEL1=0.
      DEL2=0.
      DEL3=0.
      DEL4=0.
C                        CHECK IF PANEL I IS A WING TIP PANEL.
      IF(I.GT.(NP-M)) GO TO 300
C
C                        SINGULARITIES DETERMINED IN THIS SECTION
C                        ARE FOR THE INTERIOR OF THE PLANFORM.
C
      IF(N9.NE.MC) GO TO 110
      Y3=Y(I+M,2)
      X5=X(I+M,3)/AMCH
      DEL1=(X3-X1)/(Y2-Y1)*SG(G1)
      DEL3=(X5-X3)/(Y3-Y2)*SG(G3)
      GO TO 120
  110 CONTINUE
      DEL1=SG(D1)
      DEL3=SG(D3)
  120 CONTINUE
      DEL2=SG(D2)
      DEL4=SG(D4)
```

```
      GAM1=SG(G1)
      GAM3=SG(G3)
      IF(N9.EQ.MC) GO TO 400
      GAM2=SG(G2)
      GAM4=SG(G4)
      GO TO 400
  300 CONTINUE
C
C                       SINGULARITIES DETERMINED IN THIS SECTION
C                       ARE FOR TIP CHORD PANELS.
C
      IF(N8.NE.MC) GO TO 310
      DEL1=(X3-X1)/(Y2-Y1)*SG(G1)
      GO TO 320
  310 CONTINUE
      DEL1=SG(D1)
      DEL3=SG(D3)
  320 CONTINUE
      DEL2=SG(D2)
      DEL4=SG(D4)
      GAM1=SG(G1)
      IF(N9.EQ.MC) GO TO 400
      GAM2=SG(G2)
  400 CONTINUF
C
      AL=(Y2*(X3-X4)*GAM1+(Y1*X4-X1*Y2)*GAM3+(X1*Y2-Y1*X3)*GAM4)
     1/(Y2-Y1)/(X3-X4)
C
      BL=(GAM3-GAM4)/(X3-X4)
C
      CL=(-(X3-X4)*GAM1+(X1-X4)*GAM3+(X3-X1)*GAM4)/(Y2-Y1)/(X3-X4)
C
      AT=(Y1*(X2-X1)*GAM4+(Y2*X1-X4*Y1)*GAM2+(X4*Y1-Y2*X2)*GAM1)
     1/(X2-X1)/(Y1-Y2)
C
      BT=(GAM2-GAM1)/(X2-X1)
C
      CT=(-(X2-X1)*GAM4+(X4-X1)*GAM2+(X2-X4)*GAM1)/(Y1-Y2)/(X2-X1)
C
C                       CALCULATE LIFT AND MOMENT INCREMENTS
C
      X5=X1+(X3-X1)*(YI-Y1)/(Y2-Y1)
      X6=X1+(X4-X1)*(YI-Y1)/(Y2-Y1)
      X7=X2+(X4-X2)*(YI-Y1)/(Y2-Y1)
C
      IF(N8.NE.MC) GO TO 410
      XLE=X5
  410 CONTINUE
C
      CLL(I)=2*(AL+CL*YI)*(X6-X5)+BL*(X6**2-X5**2)
      CLL(I)=CLL(I)+2*(AT+CT*YI)*(X7-X6)+BT*(X7**2-X6**2)
C
      CM(I)=(AL+CL*YI)*(X6**2-X5**2)+2*BL*(X6**3-X5**3)/3
      CM(I)=CM(I)+(AT+CT*YI)*(X7**2-X6**2)+2*BT*(X7**3-X6**3)/3
     1-XLE*CLL(I)
C
C
```

```
C                         DETERMINE IF UPPER OR LOWER PANEL VOR-
C                         TICITY EQUATIONS ARE TO BE USED
C
C                         CALCULATE THE X CO-ORDINATE OF THE PANEL
C                         MAIN DIAGONAL AT YI.
      XD=X1+(X4-X1)/(Y2-Y1)*(YI-Y1)
C                         CHECK XI TO DETERMINE IF IT LIES ABOVE
C                         OR BELOW THE MAIN DIAGONAL. NOTE - IF
C                         XI.GT.XD, THEN IT LIES BELOW THE MAIN
C                         DIAGONAL.
      IF(XI.GT.XD) GO TO 450
C
C                         UPPER PANEL EQUATIONS ARE IN THIS SECTION.
      EL=(DEL1-DEL3+(X1-X3)*CL)/(Y1-Y2)
C
      FL=(-Y2*DEL1+Y1*DEL3+(Y1*X3-X1*Y2)*CL)/(Y1-Y2)
C
      GAM(I)=AL+BL*XI+CL*YI
      DEL(I)=FL-CL*XI+EL*YI
      GO TO 500
  450 CONTINUE
C
C                         LOWER PANEL EQUATIONS ARE IN THIS SECTION
C
C
      ET=(DEL2-DEL4+(Y2-X4)*CT)/(Y1-Y2)
C
      FT=(-Y2*DEL2+Y1*DEL4+(Y1*X4-X2*Y2)*CT)/(Y1-Y2)
C
      GAM(I)=AT+BT*XI+CT*YI
      DEL(I)=FT-CT*XI+ET*YI
  500 CONTINUE
  600 CONTINUE
C
C                         CALCULATE UPPER AND LOWER SURFACE CPS.
C
      IF(NPF.EQ.1) GO TO 604
C                         EXACT ISENTROPIC PRESSURE EXPRESSION.
      DUM1=2/G/MACH**2
      DUM2=(G-1)*MACH**2/2
      DUM3=G/(G-1)
      DO 601 I=1,NP
      DUM4=(SUM(I))**2+(GAM(I)/2)**2+(DEL(I)/2)**2
      CPU(I)=DUM1*((1-DUM2*(GAM(I)+DUM4))**DUM3-1)
      CPL(I)=DUM1*((1-DUM2*(-GAM(I)+DUM4))**DUM3-1)
  601 CONTINUE
      GO TO 606
  604 CONTINUE
C                         LINEARIZED PRESSURE EXPRESSION.
      DUM1=(1-MACH**2)
      DO 605 I=1,NP
      DUM2=DUM1*(GAM(I)/2)**2+(SUM(I))**2+(DEL(I)/2)**2
      CPU(I)=-(GAM(I)+DUM2)
      CPL(I)=-(-GAM(I)+DUM2)
  605 CONTINUE
  606 CONTINUE
C
```

```
C                    START OUTPUT SEQUENCE
C
      WRITE(6,610) ALPHA
  610 FORMAT(/////29X,*ALPHA =*,F6.2)
      WRITE(6,620)
  620 FORMAT(//12X,*PRESSURE AND SINGULARITY STRENGTH DISTRIBUTION*)
      WRITE(6,625) CX,CY
  625 FORMAT(/30X,*XP =*,F6.3/30X,*YP =*,F6.3)
      IF(NPF.EQ.1) GO TO 627
      WRITE(6,626)
  626 FORMAT(/33X,*EXACT*)
      GO TO 629
  627 CONTINUE
      WRITE(6,628)
  628 FORMAT(/32X,*LINEAR*)
  629 CONTINUE
      N1=1
      DO 760 J=1,N
      DUM=S(J)+CY*(S(J+1)-S(J))
      WRITE(6,630) DUM
  630 FORMAT(//30X,*Y/S =*,F6.3/ 6X,*X/C*,4X,*CAMBER*,5X,*DEL*,6X,*GAM*,
     16X,*CPU*,6X,*CPL*,6X,*DELCP*)
      DO 750 I=1,M
      DUM1=C(I)+CX*(C(I+1)-C(I))
      DUM2=CPL(N1)-CPU(N1)
      WRITE(6,640) DUM1,CBR(N1),DEL(N1),GAM(N1),CPU(N1),CPL(N1),DUM2
  640 FORMAT(3X,F6.3,3X,F6.2,2X,F8.5,3(1X,F8.5),2X,F8.5)
      N1=N1+1
  750 CONTINUE
  760 CONTINUE
C
C                    THIS SECTION OF THE OUTPUT TABULATES THE
C                    LOADS SUMMARY WHICH INCLUDES CL AND CM.
C
      WRITE(6,765)
  765 FORMAT(//29X,*LOADS SUMMARY*//17X,*YS*,5X,*X-CP*,4X,*CHORD*,
     16X,*CL*,6X,*CM*)
      NB1=NB-1
      N4=0
      DO 790 K=1,NB1
      N1=NL(K)
      N2=NL(K+1)-1
      N3=NL(K+1)
      DO 780 J=N1,N2
      DUM=S(J)+CY*(S(J+1)-S(J))
      DUM1=(DUM-S(N1))/(S(N3)-S(N1))
      DUM2=(B(K,2)-B(K,1))*(1-DUM1)+DUM1*(B(K+1,2)-B(K+1,1))
      DUM3=0.
      DUM4=0.
      DO 770 I=1,M
      N4=N4+1
      DUM3=DUM3+CLL(N4)
      DUM4=DUM4+CM(N4)
  770 CONTINUE
      DUM5=DUM4/DUM3/DUM2
      DUM3=DUM3/DUM2
      DUM4=DUM4/(DUM2**2)
```

```
      WRITE(6,775) DUM,DUM5,DUM2,DUM3,DUM4
  775 FORMAT(14X,F5.2,3X,F5.2,2X,F7.2,2X,F7.2,2X,F6.3)
  780 CONTINUE
  790 CONTINUE
  800 CONTINUE
      STOP
      END
```

Vita

John Charles Sparks was born on 24 October 1947 in Elwood, Indiana. He graduated from high school in Xenia, Ohio, in 1965 and attended Wright State University in Dayton, Ohio. He received the degrees of Bachelor of Science (Mathematics) in June of 1969 and Master of Science (Mathematics) in June of 1971. In January of 1974, he accepted employment as a Mathematician for the Air Force Flight Dynamics Laboratory at Wright-Patterson Air Force Base, Ohio. He is currently assigned to the Aeronautical Systems Division (ASD/ENFSL) as an Aerospace Engineer.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
Paneling methods are approximate techniques for solving flow problems over wings and bodies. Vortex panels are used to model flow over wings and other lifting surfaces. The author develops a triangular vortex panel having a vorticity distribution that can vary in magnitude and direction. This panel is used to predict the pressure distribution on a rectangular and a sweptback wing in subsonic flow. Lift distributions obtained compare favorably to Anderson's solution and wind tunnel results except near the wing tip. In this region, the

Block 20. ABSTRACT

distribution will spike before satisfying the Kutta condition imposed at the tip. Possible remedies for the tip problem are discussed.